D | S | T | Q | Q | S | S

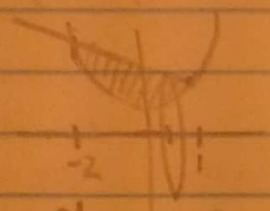
Ex: (EsEs)

L é a região limitada por $f(x) = -x+3$, $g(x) = x^2+1$

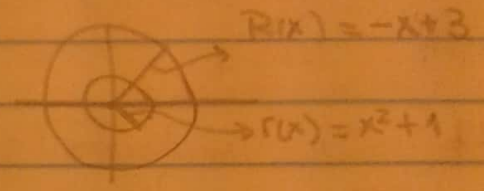
S_1 é o sólido gerado pela rotação de L em torno $y=0$

S_2 " " " " " " $y=1$

S_1:



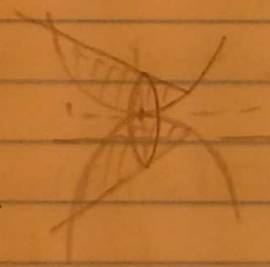
$A_1(y)$



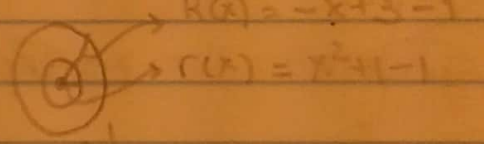
$$V_1 = \pi \int_{-2}^{1} (-x+3)^2 - (x^2+1)^2 \, dx =$$

$$= \pi \int_{-2}^{1} x^2 - 6x + 9 - x^4 - 2x^2 - 1 \, dx = \pi \int_{-2}^{1} -x^4 - x^2 - 6x + 8 \, dx =$$

S_2



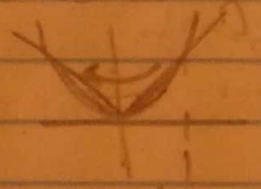
$A_2(x)$



$$V_2 = \pi \int_{-2}^{1} A_2(x) \, dx = \pi \int_{-2}^{1} (-x+2)^2 - (x^2)^2 \, dx = \checkmark$$

Ex: L região limitada por $f(x) = x^2$ e $g(x) = x$

S sólido gerado pela rotação de L em torno ao eixo y



$$A(y) = \pi \left((\sqrt{y})^2 - y^2 \right)$$

$$V = \pi \int_0^1 y - y^2 \, dy = \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1 = \pi \left(\frac{1}{2} - \frac{1}{3} \right)$$

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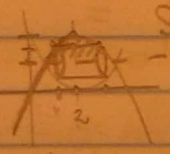
$\pm\sqrt{-y+2}+2 = x \Rightarrow 2-x = \mp\sqrt{-y+2}$

D	S	T	Q	Q	S	S

Ex: L: limitada por $f(x) = -(x-2)^2+2$ e $y \geq 1$

S: rotação em torno de $y = 1$



$$C(y) = 2\pi R(x) \cdot h(x) = 2\pi(y-1)\left[2(2-x)\right]$$

$$= 2\pi(y-1)\left[2\sqrt{-y+2}\right]$$

$2-y = u$ $\quad 2-u = y$

$-dy = du$

$$V = \int_1^2 2\pi(y-1)\,2\sqrt{-y+2}\,dy = 4\pi\int_1^2 (y-1)\sqrt{2-y}\,dy \overset{\downarrow}{=} -4\pi\int(1-u)\sqrt{u}\,du$$

$$= -4\pi\left(\int u^{1/2} - u^{3/2}\,du\right) = -4\pi\left(u^{3/2}\cdot\frac{2}{3} - \frac{2}{5}u^{5/2}\right) = -4\pi\left((2-y)^{3/2}\cdot\frac{2}{3} - \frac{2}{3}(2-y)^{5/2}\right)\Big|_1^2$$

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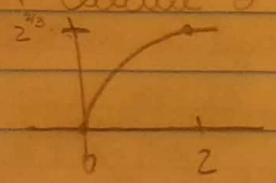
EX II: Calcule o comprimento de $f(x) = x^{2/3}$ $0 \leq x \leq 2$



OBS: $f'(x) = \frac{2}{3} x^{-1/3}$ $\nexists f'(0)$!

Então trabalho em y: $y = x^{2/3}$ $0 \leq x \leq 2$ $\rightarrow$ $g(y) = y^{3/2}$ $0 \leq y \leq 2^{2/3}$

$$L = \int_0^{2^{3/2}} \sqrt{1 + (g'(y))^2}\, dy = \int_0^{2^{3/2}} \sqrt{1 + \left(\frac{3}{2} y^{1/2}\right)^2}\, dy = \int_0^{2^{3/2}} \sqrt{1 + \frac{9}{4} y}\, dy$$

$$= (1 + 9/4\, y)^{3/2} \cdot \frac{2}{3} \cdot \frac{4}{9} \Big|_0^{2^{3/2}} = \frac{8}{27} \left(1 + \frac{9}{4} \overset{2 \cdot 2^2}{\sqrt{8}} - 1\right) = \frac{8}{27} \cdot \frac{9}{4}\, 2\sqrt{2} = \frac{4\sqrt{2}}{3}$$

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