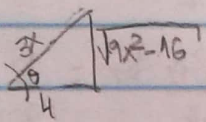
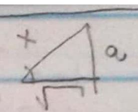
EX: $\int \frac{dx}{\sqrt{9x^2-16}}$



$\cos\theta = \frac{4}{3x}$

$\tan\theta = \sqrt{9x^2-16}/4$

OBS:



$\operatorname{sen}\theta = \frac{a}{x}$

$\tan\theta = \frac{a}{\sqrt{\ }}$

$\therefore \int \frac{d\theta}{\operatorname{sen}\theta}$

$x = \frac{4}{3\cos\theta}$

$dx = \frac{+4\operatorname{sen}\theta\, d\theta}{3\cos^2\theta}$

$$\int \frac{dx}{\sqrt{9x^2-16}} = \int \frac{1}{4\tan\theta} \cdot \frac{4\operatorname{sen}\theta}{3\cos^2\theta}\, d\theta = \frac{1}{3}\int \frac{d\theta}{\cos\theta} = \frac{1}{3}\ln\left(\frac{1}{\cos\theta} + \tan\theta\right)$$

$$= 1/3 \ln\left(3x/4 + \sqrt{9x^2-16}/4\right) + C$$

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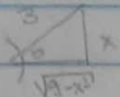
Ex. $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$



$x = 3\,\text{sen}\,\theta$ $\quad \sqrt{9-x^2} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

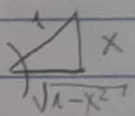
$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int \frac{9\,\text{sen}^2\theta}{3\cos\theta}\cdot 3\cos\theta\, d\theta = 9\int \text{sen}^2\theta\, d\theta = 9\int \frac{1-\cos(2\theta)}{2}\, d\theta =$$

$$= 9\left(\frac{\theta}{2} - \frac{\text{sen}(2\theta)}{4}\right) + C = \frac{9}{2}\left(\theta - \text{sen}\,\theta\cos\theta\right) + C =$$

$$= 9/2\left(\text{arcsen}\left(\frac{x}{3}\right) - \frac{x}{3}\cdot\sqrt{9-x^2}\right) + C$$

MAIS EXEMPLOS:

1) Voltar ao exemplo em substituições:



$$\int \sqrt{1-x^2}\,dx = \int \sqrt{1-\text{sen}^2\theta}\,\cos\theta\, d\theta = \int \cos^2\theta\, d\theta = \int \frac{\cos(2\theta)+1}{2}\, d\theta =$$

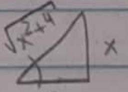
$x = \text{sen}\,\theta$

$dx = \cos\theta\, d\theta$

$$= +\frac{\text{sen}(2\theta)}{4} + \frac{\theta}{2} + C = \frac{1}{2}(\text{sen}\,\theta\cos\theta + \theta) + C$$

$$= \frac{1}{2}\left(x\sqrt{1-x^2} + \text{arcsen}\,x\right) + C$$

2) $\int \frac{dx}{(x^2+4)^2}$ $\quad dx = 2/\cos^2\theta\, d\theta$ $\quad x = 2\tan\theta$



$\tan\theta = \frac{x}{2}$

$\cos\theta = 2/\sqrt{x^2+4}$

$$\therefore \int \frac{dx}{(x^2+4)^2} = \int \left(\frac{\cos\theta}{2}\right)^4 \cdot \frac{2\,d\theta}{\cos^2\theta} = \int \frac{\cos^2\theta\, d\theta}{2^3} = \frac{1}{8}\int \frac{\cos(2\theta)+1}{2}\, d\theta =$$

$$= 1/8\left(\text{sen}(2\theta)/4 + \theta/2\right) + C = 1/16\left(\text{sen}\,\theta\cos\theta + \theta\right) + C$$

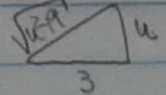
$$= 1/16\left(\frac{x}{\sqrt{x^2+4}}\cdot\frac{2}{\sqrt{x^2+4}} + \arctan\left(\frac{x}{2}\right)\right) + C$$

3) $\int_0^{\ln 4} \frac{e^t}{\sqrt{e^{2t}+9}}\,dt \underset{u=e^t}{=} \int_1^4 \frac{du}{\sqrt{u^2+9}} = \int$

$du = e^t dt$ $\quad u = 3\tan\theta$

$0 < t < \ln 4$

$1 < u < 4$



$\cos\theta = 3/\sqrt{\ }$

$\tan\theta = \frac{u}{3}$

$\frac{d\theta}{\cos^2\theta} = \frac{du}{3}$

$$\int \frac{du}{\sqrt{u^2+9}} = \int \frac{\cos\theta}{3}\cdot\frac{3\,d\theta}{\cos^2\theta} = \int \frac{d\theta}{\cos\theta} = \ln\left(\frac{1}{\cos\theta} + \tan\theta\right) + C = \ln\left(\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right) + C$$

$$= \ln\left(\sqrt{e^{2t}+9}/3 + e^t/3\right) + C$$

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