CALCULO 2A - TURMA B1 -

1) Pontos críticos de $g(x) = \int_{-1}^{x^3} \arctan(t-1)e^{3t}\,dt \quad x \geq -1$

Se $F(t)$ é a primitiva de $\arctan(t-1)e^{3t}$, então

$g(x) = F(x^3) - F(-1)$

$\therefore g'(x) = (F(x^3))' - 0 = \arctan(x^3-1)e^{3x^3} \cdot 3x^2$

$g'(x) = 0 \iff \arctan(x^3-1)e^{3x^3}3x^2 = 0 \iff \arctan(x^3-1)3x^2 = 0$

$x = 1$ ou $x = 0$

$\therefore$ Pontos críticos $\{1, 0\}$.

2) a) $\int_0^{\pi} \underset{g}{3^x} \underset{f'}{\cos x}\,dx = 3^x \operatorname{sen} x \Big|_0^{\pi} - \int_0^{\pi} \underset{g}{3^x \ln 3} \underset{f'}{\operatorname{sen} x}\,dx =$

$= 3^x \operatorname{sen} x \Big|_0^{\pi} - \ln 3 \left(-3^x \cos x \Big|_0^{\pi} + \int_0^{\pi} 3^x \ln 3 \cos x\,dx\right)$

$= 3^x \operatorname{sen} x + \ln 3 \cdot 3^x \cos x \Big|_0^{\pi} - (\ln 3)^2 \int_0^{\pi} 3^x \cos x\,dx$

$\therefore (1+(\ln 3)^2)\int_0^{\pi} 3^x \cos x\,dx = 3^x \operatorname{sen} x + \ln 3 \cdot 3^x \cos x \Big|_0^{\pi}$

$$\int_0^{\pi} 3^x \cos x\,dx = \frac{3^x \operatorname{sen} x + (\ln 3)3^x \cos x}{1+(\ln 3)^2}\Bigg|_0^{\pi} = \frac{-(\ln 3)3^{\pi} - \ln 3}{1+(\ln 3)^2} = -\frac{(\ln 3)(3^{\pi}+1)}{1+(\ln 3)^2}$$

b) $\int \frac{x\,dx}{(4x^2+3)(x-1)} \rightarrow$ Frações parciais:

$$\frac{x}{(4x^2+3)(x-1)} = \frac{Ax+B}{(4x^2+3)} + \frac{C}{x-1} = \frac{\overbrace{(Ax+B)(x-1)}^{Ax^2-Ax+Bx-B} + C(4x^2+3)}{(4x^2+3)(x-1)} = \frac{x^2(A+4C)+x(-A+B)-B+3C}{(4x^2+3)(x-1)}$$

$$\Rightarrow \begin{cases} A+4C=0 & \Rightarrow A=-4C \\ -A+B=1 & \Rightarrow -4C+3C=1 \Rightarrow -C=1 \\ -B+3C=0 & \Rightarrow B=3C \end{cases}$$

$C=1$

$A=-4$
$B=3$
$C=1$

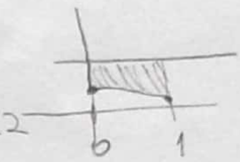
$$\therefore \frac{x}{(4x^2+3)(x-1)} = \frac{-4x}{4x^2+3} + \frac{3}{4x^2+3} + \frac{1}{x-1}$$

$$\int \frac{x\,dx}{(4x^2+3)(x-1)} = -4\int \frac{x\,dx}{4x^2+3} + 3\underbrace{\int \frac{dx}{4x^2+3}}_{(*)} + \int \frac{dx}{x-1}$$

$$(*)\ \int \frac{dx}{4x^2+3} = \frac{1}{3}\int \frac{dx}{\frac{4}{3}x^2+1} = \frac{1}{3}\int \frac{dx}{\left(\frac{2}{\sqrt{3}}x\right)^2+1} = \frac{1}{3}\cdot\frac{\sqrt{3}}{2}\arctan\left(\frac{2}{\sqrt{3}}x\right)+C$$

$$\therefore \int \frac{x\,dx}{(4x^2+3)(x-1)} = -\frac{1}{2}\ln(4x^2+3) + \frac{\sqrt{3}}{2}\arctan\left(\frac{2}{\sqrt{3}}x\right) + \ln(x-1) + C$$

3) a) Área entre curvas: $f(x) = 2$, $g(x) = \frac{1}{\sqrt{16+4x^2}} < 2$



$$A = \int_0^1 \left(2 - \frac{1}{\sqrt{16+4x^2}}\right)dx = \int_0^1 2\,dx - \underbrace{\int_0^1 \frac{dx}{\sqrt{16+4x^2}}}_{(*)}$$

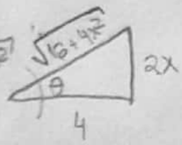
$$(*)\ \int \frac{dx}{\sqrt{16+4x^2}} \underset{\uparrow}{=} \int \frac{\cos\theta}{4}\cdot\frac{2}{\cos^2\theta}\,d\theta = \frac{1}{2}\int\frac{d\theta}{\cos\theta} = \frac{1}{2}\int \frac{\cos\theta}{\cos^2\theta}\,d\theta =$$

$x = 2\tan\theta$

$dx = \frac{2}{\cos^2\theta}\,d\theta$

$\cos\theta = \frac{4}{\sqrt{16+4x^2}}$

$\tan\theta = \frac{2x}{4}$



$u = \operatorname{sen}\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{2}\int \frac{\cos\theta\,d\theta}{1-\operatorname{sen}^2\theta} = \frac{1}{2}\int\frac{du}{1-u^2} = \frac{1}{2}\int\frac{du}{(1+u)(1-u)} = \frac{1}{4}\int\frac{du}{1+u} + \frac{1}{4}\int\frac{du}{1-u}$$

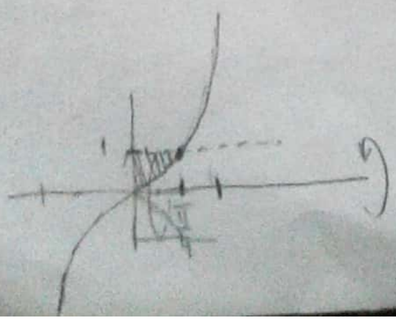
$$= \frac{1}{4}\left(\ln\left(\frac{1+u}{1-u}\right)\right)+C = \frac{1}{4}\ln\left(\frac{1+\operatorname{sen}\theta}{1-\operatorname{sen}\theta}\right)+C = \frac{1}{2}\ln\sqrt{\frac{(1+\operatorname{sen}\theta)^2}{1-\operatorname{sen}^2\theta}}+C = \frac{1}{2}\ln\left(\frac{1+\operatorname{sen}\theta}{\cos\theta}\right)+C$$

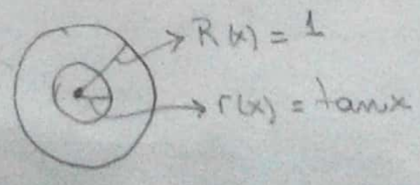
$$(*)\ \frac{1}{1-u^2} = \frac{A}{1+u} + \frac{B}{1-u} = \frac{A(1-u)+B(1+u)}{1-u^2} = \frac{u(B-A)+A+B}{1-u^2} \Rightarrow \begin{matrix} B-A=0 \\ A+B=1 \end{matrix} \quad A = B = \frac{1}{2}$$

$$\therefore \int_0^1\left(2 - \frac{1}{\sqrt{16-4x^2}}\right)dx = 2x\Big|_0^1 - \frac{1}{2}\ln\left(\frac{\sqrt{16+4x^2}}{4}+\frac{2x}{4}\right)\Big|_0^1 = 2 - \frac{1}{2}\ln\left(\frac{\sqrt{20}}{4}+\frac{1}{2}\right) + 0$$

A área total é $2 - \frac{1}{2}\ln\left(\frac{\sqrt{20}}{4}+\frac{1}{2}\right)$

b) L





• Seção transversal: $A(x) = \pi R^2(x) - \pi r^2(x) = \pi - \pi \tan^2 x$

• $V = \int_0^{\pi/4} A(x)\,dx = \int_0^{\pi/4} (\pi - \pi \tan^2 x)\,dx = \int_0^{\pi} \left(\pi - \pi \frac{\operatorname{sen}^2 x}{\cos^2 x}\right) dx = \int_0^{\pi/4} \left(\pi - \pi\left(\frac{1-\cos^2 x}{\cos^2 x}\right)\right) dx =$

$= \int_0^{\pi/4} \left(\pi - \frac{\pi}{\cos^2 x} + \pi\right) dx = \int_0^{\pi/4} 2\pi\,dx - \pi \int_0^{\pi/4} \frac{dx}{\cos^2 x} = 2\pi x \Big|_0^{\pi/4} - \pi \tan x \Big|_0^{\pi/4} = \frac{2\pi^2}{4} - \pi$

O volume é $\frac{\pi^2}{2} - \pi$

4) Convergência:

a) $\int_2^{\infty} \frac{\sqrt{e^{3x}+5}}{e^{4x}}\,dx$

Comparo $\frac{\sqrt{e^{3x}+5}}{e^{4x}}$ com $\frac{e^{\frac{3}{2}x}}{e^{4x}} = e^{-\frac{5}{2}x} = \frac{1}{e^{5/2 x}}$

Verifico com o limite:

$\lim_{x\to\infty} \frac{\sqrt{e^{3x}+5}}{e^{4x}} \Big/ \frac{1}{e^{5/2 x}} = \lim_{x\to\infty} \frac{e^{\frac{3}{2}x}\sqrt{1+e^{-3x}}}{e^{4x}} \cdot e^{\frac{5}{2}x} = 1 \neq 0$

Como $\frac{\sqrt{e^{3x}+5}}{e^{4x}}$ e $\frac{1}{e^{5/2 x}}$ tem o mesmo comportamento no infinito, então a convergência de $\int_2^{\infty} \frac{dx}{e^{5/2 x}}$ garante a convergencia de $\int \frac{\sqrt{e^{3x}+5}}{e^{4x}}\,dx$

• $\int_2^{\infty} \frac{dx}{e^{\frac{5x}{2}}} = \lim_{a\to\infty} \int_2^a e^{-\frac{5}{2}x}\,dx = \lim_{a\to\infty} -e^{-\frac{5}{2}x} \cdot \frac{2}{5} \Big|_2^a = \lim_{a\to\infty} \frac{2}{5}\left(-e^{-\frac{5a}{2}} + e^{-5}\right) = \frac{2}{5} e^{-5}$

b) $\int_0^2 (\ln x)^2\,dx = \lim_{a\to 0^+} \int_a^2 \ln^2 x\,dx = \lim_{a\to 0^+} x\ln^2 x \Big|_a^2 - \int_a^2 x \cdot 2\ln x \cdot \frac{1}{x}\,dx = \lim_{a\to 0^+} x\ln^2 x - 2\int_a^2 \ln x\,dx$

$= \lim_{a\to 0^+} x\ln^2 x \Big|_a^2 - 2\left(x\ln x \Big|_a^2 - \int_a^2 x \cdot \frac{1}{x}\,dx\right) = \lim_{a\to 0^+} x\ln^2 x - 2x\ln x + 2x \Big|_a^2 =$

$= \lim_{a\to 0^+} 2\ln^2 2 - 4\ln 2 + 4 - \underbrace{a\ln^2 a}_{\to 0} + \underbrace{2a\ln a}_{\to 0} - \underbrace{2a}_{\to 0} = 2\ln^2 2 - 4\ln 2 + 4$

A integral converge.

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