CALCULO 2A -TURMA D1-

1) a) $\displaystyle\int \frac{x\,dx}{(x+2)(5x^2+3)}$ → Frações parciais

$+3A+2C$

$$\frac{x}{(x+2)(5x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{5x^2+3} = \frac{5Ax^2+3A+Bx^2+2Bx+Cx+2C}{(x+2)(5x^2+3)} = \frac{x^2(5A+B)+x(2B+C)}{(x+2)(5x^2+3)}$$

$\therefore$ $5A+B=0 \Rightarrow B=-5A$

$2B+C=1 \quad \Rightarrow -10A-\frac{3}{2}A = 1 \Rightarrow (-20-3)A = 2$

$3A+2C=0 \Rightarrow C = -\frac{3}{2}A$

$A = \frac{-2}{23} \quad B = \frac{10}{23} \quad C = \frac{3}{23}$

$$\therefore \int \frac{x\,dx}{(x+2)(5x^2+3)} = \frac{-2}{23}\int \frac{dx}{x+2} + \frac{10}{23}\int\frac{x\,dx}{5x^2+3} + \frac{3}{23}\underbrace{\int\frac{dx}{5x^2+3}}_{(*)}$$

$$(*)\ \int\frac{dx}{5x^2+3} = \frac{1}{3}\int\frac{dx}{\frac{5x^2}{3}+1} = \frac{1}{3}\int\frac{dx}{\left(\frac{\sqrt{5}}{\sqrt{3}}x\right)^2+1} = \frac{1}{3}\frac{\sqrt{3}}{\sqrt{5}}\arctan\left(\sqrt{\frac{5}{3}}\,x\right)+C$$

$$\int\frac{x\,dx}{(x+2)(5x^2+3)} = \frac{-2}{23}\ln(x+2) + \frac{1}{23}\ln(5x^2+3) + \frac{\sqrt{3}}{23\sqrt{5}}\arctan\left(\sqrt{\frac{5}{3}}\,x\right) + C$$

b) $$\int_0^{\pi/2} 5^x \operatorname{sen} x\,dx = -5^x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} 5^x\ln 5\,\cos x\,dx = -5^x\cos x + \ln 5\left(\int_0^{\pi/2} 5^x\cos x\,dx\right) =$$

$$= -5^x\cos x\Big|_0^{\pi/2} + \ln 5\left(5^x\operatorname{sen}x\Big|_0^{\pi/2} - \int_0^{\pi/2} 5^x\ln 5\operatorname{sen}x\right) =$$

$$\therefore (1+(\ln 5)^2)\int_0^{\pi/2} 5^x\operatorname{sen}x\,dx = -5^x\cos x + \ln 5\cdot 5^x\operatorname{sen}x\Big|_0^{\pi/2}$$

$$\therefore \int_0^{\pi/2} 5^x\operatorname{sen}x\,dx = \frac{-5^x\cos x + \ln 5\cdot 5^x\operatorname{sen}x}{1+(\ln 5)^2}\Bigg|_0^{\pi/2} = \frac{\ln 5\cdot 5^{\pi/2}+1}{1+(\ln 5)^2}$$

2) Pontos críticos da função $g(x) = \displaystyle\int_{-1}^{x^2}\arctan(t-1)e^{5t}\,dt \qquad x \geq -1$

Se $G(x)$ é a primitiva de $\arctan(t-1)\,e^{5t}$ então

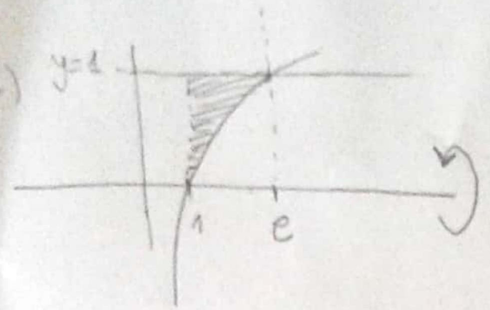
$$g(x) = \int_{-1}^{x^2}\arccos(t-1)e^{5t}\,dt = G(x^2) - G(-1)$$

$$g'(x) = G'(x^2)\cdot 2x - 0 = \arctan(x^2-1)e^{5x^2}\cdot 2x$$

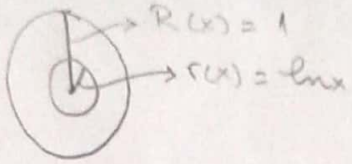
$g'(x) = 0 \iff \arctan(x^2-1)2x = 0 \iff x = 0$
$x = 1$

Ptos críticos $\{x=0, x=1\}$

3) a)



• Seção transversal



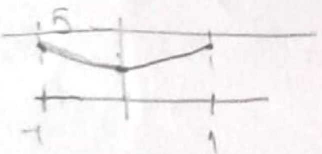
$A(x) = \pi - \pi(\ln x)^2$

• $V = \int_1^e \pi - \pi(\ln x)^2\,dx = \pi x\Big|_1^e - \pi\left(x\ln^2 x\Big|_1^e - \int_1^e x\frac{2\ln x}{x}\,dx\right) =$

$= \pi x\Big|_1^e - \pi x\ln^2 x + 2\pi\left(x\ln x\Big|_1^e - \int_1^e dx\right) = \pi\left(x - x\ln^2 x + 2x\ln x - 2x\right)\Big|_1^e =$

$= \pi(e - e + 2e - 2e) - \pi(1 - 0 + 0 - 2) = \pi$

b) Area

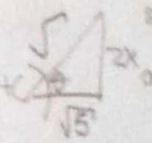


$A = \int_{-1}^{1} 5 - \sqrt{5+4x^2}\,dx$

• $\int \sqrt{5+4x^2}\,dx \underset{\uparrow}{=} \int \frac{\sqrt{5}}{\cos\theta}\cdot\frac{\sqrt{5}}{2\cos^2\theta}\,d\theta = \frac{5}{2}\underbrace{\int\frac{d\theta}{\cos^3\theta}}_{A} = \frac{5}{4}\left(\frac{\tan\theta}{\cos\theta} + \ln\left(\frac{1}{\cos\theta} + \tan\theta\right)\right) + C$

$x = \frac{\sqrt{5}}{2}\tan\theta$

$dx = \frac{\sqrt{5}}{2}\frac{d\theta}{\cos^2\theta}$



$\operatorname{sen}\theta = \frac{2x}{\sqrt{\ }}$

$\cos\theta = \frac{\sqrt{5}}{\sqrt{5+4x^2}}$

$\tan\theta = \frac{2x}{\sqrt{5}}$

$= \frac{5}{4}\left(\frac{2x}{\sqrt{5}}\frac{\sqrt{5+4x^2}}{\sqrt{5}} + \ln\left(\frac{\sqrt{5+4x^2}}{\sqrt{5}} + \frac{2x}{\sqrt{5}}\right)\right) + C$

A $\int\frac{d\theta}{\cos^3\theta} = \int\frac{d\theta}{\cos^2\theta}\frac{1}{\cos\theta} = \frac{\tan\theta}{\cos\theta} - \int\frac{\tan\theta}{\cos\theta}\operatorname{sen}\theta\,d\theta = \frac{\tan\theta}{\cos\theta} - \int\frac{\operatorname{sen}^2\theta}{\cos^3\theta}\,d\theta = \text{to}$

$= \frac{\tan\theta}{\cos\theta} - \int\frac{d\theta}{\cos^3\theta} + \int\frac{d\theta}{\cos\theta} \quad \therefore \int\frac{d\theta}{\cos^3} = \frac{1}{2}\left(\frac{\tan\theta}{\cos\theta} + \ln\left(\frac{1}{\cos\theta} + \tan\theta\right)\right) + C$

$\therefore A = \int_0^1 5 - \sqrt{5+4x^2}\, dx = 5x - \frac{5}{4}\left(\frac{2}{5}x\sqrt{5+4x^2} + \ln\left(\frac{\sqrt{5+4x^2}}{\sqrt{5}} + \frac{2x}{\sqrt{5}}\right)\right|_{-1}^{1} =$

$= 5 - \frac{5}{4}\left(\frac{2}{5}\sqrt{20} + \ln\left(\frac{\sqrt{20}}{\sqrt{5}} + \frac{2}{\sqrt{5}}\right)\right) - \left(-5 - \frac{5}{4}\left(\frac{-2}{5}\sqrt{20} + \ln\left(\frac{\sqrt{20}}{\sqrt{5}} - \frac{2}{\sqrt{5}}\right)\right)\right) =$

$= 10 - \sqrt{20} + \ln\left(\frac{\sqrt{20}}{\sqrt{5}} + \frac{2}{\sqrt{5}}\right) + \ln\left(\frac{\sqrt{20}}{\sqrt{5}} - \frac{2}{\sqrt{5}}\right)$

4) Convergência

a) $\int_2^\infty \frac{e^{2x}}{\sqrt{e^{5x}+5}}\, dx$

I) Comparação: $\frac{e^{2x}}{\sqrt{e^{5x}+5}} \sim \frac{e^{2x}}{e^{5/2x}} = \frac{1}{e^{x/2}}$

II) Verifico: $\lim_{x\to\infty} \frac{e^{2x}}{\sqrt{e^{5x}+5}} \Big/ 1/e^{x/2} = \lim_{x\to\infty} \frac{e^{x/2}.e^{2x}}{\sqrt{e^{5x}+5}} = \lim_{x\to\infty} \frac{e^{\cancel{5/2x}}}{\cancel{\sqrt{e^{5x}}}\left(\sqrt{1+5e^{-5x}}\right)} = 1$

$\therefore \frac{e^{2x}}{\sqrt{e^{5x}+5x}}$ tem o mesmo comportamento (no infinito) que $\frac{1}{e^{x/2}}$

III) $\cdot \int_2^\infty \frac{dx}{e^{x/2}} = \lim_{a\to\infty} \int_2^{\infty} e^{-x/2}\, dx = \lim_{a\to\infty} -2e^{-x/2}\Big|_2^a = \lim_{a\to\infty} -2e^{-a/2} + 2e^{-1} = 2e^{-1}$

Como $\int_2^\infty \frac{dx}{e^{x/2}}$ converge então $\int_2^\infty \frac{e^{2x}}{\sqrt{e^{5x}+5}}\, dx$ converge.

b) $\int_0^{\pi/2} \tan^2 x\, dx = \lim_{a\to\frac{\pi}{2}^-} \int_0^a \frac{\overbrace{\sin^2 x}^{1-\cos^2 x}}{\cos^2 x}\, dx = \lim_{a\to\frac{\pi}{2}^-} \int_0^a \frac{1}{\cos^2 x} - 1\, d\theta =$

$= \lim_{a\to\frac{\pi}{2}^-} \tan x \Big|_0^a - x\Big|_0^a = \lim_{a\to\frac{\pi}{2}^-} \tan a - a - (0-0) = \infty$

$\therefore$ A integral diverge.

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