Internet

EX: $\begin{cases} 3xy' - y = \ln x + 1 \\ y(1) = -2 \end{cases}$ → Eq. linear

1) Forma padrão: $y' - \dfrac{y}{3x} = \dfrac{\ln x}{3x} + \dfrac{1}{3x}$

2) Fator integrante: $v(x)y' - \underbrace{\dfrac{v(x)}{3x}}_{v'} y = v(x)\left(\dfrac{\ln x}{3x} + \dfrac{1}{3x}\right)$

$\therefore v'(x) = -v(x)/3x \Rightarrow \dfrac{dv}{v} = -\dfrac{dx}{3x} \Rightarrow \ln v = -\dfrac{\ln x}{3} + C$

$\therefore C = 0 \Rightarrow v(x) = x^{-1/3} = 1/\sqrt[3]{x}$

3) Resolvemos a eq. dif. $\dfrac{d}{dx}\left(\dfrac{y}{\sqrt[3]{x}}\right) = \dfrac{1}{x^{\frac{1}{3}}} \cdot \left(\dfrac{\ln x}{3x} + \dfrac{1}{3x}\right)$

$\therefore y(x) = \sqrt[3]{x} \int \left(\dfrac{\ln x}{3x^{2/3}} + \dfrac{1}{3x^{2/3}}\right) dx = \dfrac{\sqrt[3]{x}}{3} \int \left(x^{-2/3} \ln x + x^{-2/3}\right) dx$

$= \dfrac{\sqrt[3]{x}}{3}\left(3x^{1/3} \ln x - \int \dfrac{3x^{1/3}}{x} dx + 3x^{1/3} + C\right)$

$= \sqrt[3]{x}\left(x^{1/3} \ln x - 3x^{1/3} + x^{1/3} + C\right) =$

Sol. geral: $y(x) = x^{2/3} \ln x - 2x^{2/3} + Cx^{1/3}$

Sol. Particular: $y(1) = -2 \Rightarrow 0 - 2 + C = -2 \Rightarrow C = 0$

Sol. do PVI: $\boxed{y(x) = x^{2/3} \ln x - 2x^{2/3}}$