(I) Ex: $(xy+1)dx + x^2 dy = 0$

I) $\dfrac{\partial N(x,y)}{\partial x} = y$ $\qquad$ $\dfrac{\partial M}{\partial y} = 2x$

$\cdot$ Calculo $\partial(x)$ para que

$$a x(xy+1)dx + a x^2 \, dy$$

seja exata.

I)a $\quad \dfrac{\partial \bar{J}}{\partial x} = a' x^2 + a 2x \qquad\qquad \dfrac{\partial M}{\partial y} = a \cdot x$

$\quad \therefore \; a' x^2 + a 2x = a x \quad \Rightarrow \quad a' x^2 = -x a \quad \Rightarrow \quad \dfrac{da}{dx} = -\dfrac{1}{x} \cdot a$

$\quad \therefore \; \dfrac{da}{a} = -\dfrac{dx}{x} \qquad \therefore \; \ln a = -\ln(x)$

$\qquad\qquad\qquad\qquad \therefore \; a(x) = 1/x$

II) Resolvemos $\quad \dfrac{xy+1}{x} dx + x \, dy = 0$

1) $\dfrac{\partial M}{\partial y} = 1 \qquad\qquad \dfrac{\partial N}{\partial x} = 1$

2) $U(x,y) = \int M(x,y)\,dx + G(y) = \int \left(y + \dfrac{1}{x}\right)dx + G(y) = v$
$\qquad\qquad = yx + \ln x + G(y)$

$\dfrac{\partial U}{\partial y} = N(x,y) \quad \Rightarrow \quad x + G'(y) = x \quad \Rightarrow \quad G'(y) = 0 \Rightarrow G(y) = C$

$\quad \therefore \; U(x,y) = xy + \ln x + C$

Sol. geral : $\{(x,y) : U(x,y) = c\} \Leftrightarrow y(x) = -\dfrac{\ln x}{x} + \dfrac{C}{x}$

OBS: O fator a pode depender de y ou de (x,y)
$\qquad\qquad A = a(y) \qquad\quad a = a(x,y)$