• EXEMPLOS:

1) $x^2 y'' - 3x y' - y = 0$ [raízes em $\mathbb{R}$ ≠]

• Proposta de solução: $y(x) = x^m$ $\quad y_1'(x) = m x^{m-1}$ $\quad y_1'' = m(m-1)x^{m-2}$

$0 = x^m[m(m-1) - 3m - 1] = x^m[m^2 - 4m - 1]$

$\therefore m = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \frac{\sqrt{20}}{2} = 2 \pm \sqrt{5}$ $\quad (\sqrt{5 \cdot 4})$

Conj. fund. $\{ y_1(x) = x^{2+\sqrt{5}},\ y_2(x) = x^{2-\sqrt{5}} \}$

2) $x^2 y'' + 3xy' + 3y = 0$ [Raízes em $\mathbb{C}$]

• Proposta de solução $y_1(x) = x^m$

$0 = x^m(m(m-1) + 3m + 3) = x^m(m^2 + 2m + 3)$

$m = \frac{-2 \pm \sqrt{4 - 4.3}}{2} = -1 \pm \frac{\sqrt{-8}}{2} = -1 \pm i\sqrt{2}$ $\quad (2 \cdot 4)$

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Soluções em $\mathbb{C}$: $y_1(x) = x^{-1+i\sqrt{2}} = x^{-1} \cdot e^{i\sqrt{2}\ln x} = \frac{1}{x}(\cos(\sqrt{2}\ln x) + i\,\text{sen}(\sqrt{2}\ln x))$

$y_2(x) = x^{-1-i\sqrt{2}} = x^{-1} \cdot e^{-i\sqrt{2}\ln x} = \frac{1}{x}(\cos(\sqrt{2}\ln x) - i\,\text{sen}(\sqrt{2}\ln x))$

Soluções em $\mathbb{R}$:

$z_1(x) = \frac{y_1+y_2}{2} = \frac{1}{x}\cos(\sqrt{2}\ln x)$, $z_2(x) = \frac{y_1-y_2}{2i} = \frac{1}{x}\text{sen}(\sqrt{2}\ln x)$

Conj. fund. $\left\{ z_1(x) = \frac{\cos(\sqrt{2}\ln x)}{x},\ z_2(x) = \frac{\text{sen}(\sqrt{2}\ln x)}{x} \right\}$

3) $4x^2y'' + 8xy' + y = 0$ [Raízes repetidas]

Proposta de solução: $y(x) = x^m$

$\therefore 0 = x^m(4m(m-1) + 8m + 1) = x^m(4m^2 + 4m + 1)$

$$m = \frac{-4 \pm \sqrt{16 - 4.4}}{2.4} = -1/2$$

$\therefore\ \cdot\ y_1(x) = x^{-1/2} = 1/\sqrt{x}$

$\cdot\ y_2(x) \rightarrow$ Redução da ordem: $y_2(x) = u(x)\,y_1(x)$

① $y_2' = u'y_1 + uy_1'$ $y_2'' = u''y_1 + uy_1'' + 2u'y_1'$

$0 = 4x^2(u''y + 2u'y_1' + uy_1'') + 8x(u'y_1 + uy_1') + uy_1 =$
$= u(\qquad) + u'(8x^2y_1' + 8xy_1) + u''4x^2y_1$

$\therefore\ 0 = u'8\left(x^2\left(\frac{-1}{2}\right)x^{-3/2} + x\,x^{-1/2}\right) + u''4x^2x^{-1/2}$

$u' = w \rightarrow 0 = w8\left(-\frac{1}{2}x^{1/2} + x^{1/2}\right) + w'4x^{3/2}$

$0 = +w\,x^{1/2} + w'\,x^{3/2}$

$-\frac{x^{1/2}}{x^{3/2}}dx = \frac{dw}{w} \Rightarrow -\frac{dx}{x} = \frac{dw}{w}$

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$\therefore -\ln x = \ln w \Rightarrow 1/x = w \Rightarrow \ln x = u(x)$

$\therefore y_2(x) = \dfrac{\ln x}{\sqrt{x}}$ $\quad \therefore$ Cony. fund. $\left\{ 1/\sqrt{x},\ \ln x/\sqrt{x} \right\}$

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