1) $P(t)$ é a população da terra $\longrightarrow$ $P'(t) = kP(t)$

$P(0) = 6 \cdot 10^8$

$P(300) = 2{,}8 \cdot 10^9$

$\rightarrow$ Resolução $P'(t) = kP(t)$

(separação de variáveis) $P(t) = \underbrace{A}_{\text{constante arbitrária}} e^{kt}$

k $\rightarrow$ const. de proporc.

$\bullet\ P(0) = A = 6 \cdot 10^8 \longrightarrow P(t) = 6 \cdot 10^8 e^{kt}$

$\bullet\ P(300) = 2{,}8 \cdot 10^9 = 6 \cdot 10^8 e^{k300}$

$28 = 6 \cdot e^{k300} \Rightarrow \frac{14}{3} = e^{k300} \Rightarrow \ln\left(\frac{14}{3}\right) = k300 \Rightarrow \frac{\ln\left(\frac{14}{3}\right)}{300} = k$

$$\boxed{P(t) = 6 \cdot 10^8 \cdot e^{\frac{\ln\left(\frac{14}{3}\right)}{300} \cdot t}}$$

Sep. variaveis $P'(t) = kP(t) \Rightarrow \frac{dP}{dt} = kP \Rightarrow \frac{dP}{P} = k\,dt$

$\int \frac{dP}{P} = \int k\,dt \Rightarrow \ln P = kt + C$

$e^{\ln P} = e^{kt+c} = e^{kt} \cdot e^{c} \;{=A} \Rightarrow P(t) = A \cdot e^{kt}$

- $P(t) = 2{,}4 \cdot 10^{10}$

$P(t) = 6 \cdot 10^{8} \cdot e^{\frac{\ln\left(\frac{14}{3}\right)}{300} t} = 2{,}4 \cdot 10^{10}$

$6 \cdot e^{\frac{\ln\left(\frac{14}{3}\right)}{300} t} = 240$

$\frac{\ln\left(\frac{14}{30}\right) t}{300} = \ln(40) \Rightarrow t = \frac{\ln(40) \cdot 300}{\ln(14/3)}$

Rta: ano: $1650 + t$

✓

2 _ a) PVI $\begin{cases} x y' = \ln x - 2y \\ y(1) = 1/4 \end{cases}$

• Eq. linear: $y' + \frac{2}{x} y = \frac{\ln x}{x}$

→ Fator integrante: $\underbrace{v y' + \overbrace{v \frac{2}{x}}^{v'} y}_{(v y)'} = v \frac{\ln x}{x}$

$v' = \frac{2}{x} v$ (variáveis sep.)

$\frac{dv}{dx} = \frac{2}{x} v \Rightarrow \frac{dv}{v} = \frac{2\,dx}{x} \Rightarrow \int \frac{dv}{v} = \int \frac{2\,dx}{x}$

$\boxed{v(x) = x^2}$

$\ln v = 2 \ln x \Rightarrow \ln v = \ln x^2 \Rightarrow \boxed{v(x) = x^2}$

$$(v y)' = v \frac{\ln x}{x} \Rightarrow (x^2 y)' = x^2 \frac{\ln x}{x} \Rightarrow x^2 y = \underbrace{\int x \ln x \, dx}_{A}$$

$$A \quad \int \underbrace{x}_{g'} \underbrace{\ln x}_{f} \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\therefore \; y(x) = \frac{1}{x^2}\left(\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C \right) = \boxed{\frac{\ln x}{2} - \frac{1}{4} + \frac{C}{x^2} = y(x)}$$ Sol. geral

- Sol. do PVI.

$$y(1) = 1/4 \rightarrow y(1) = -\frac{1}{4} + C = \frac{1}{4}$$

$$\boxed{C = 1/2}$$

Solucao do PVI:

$$\boxed{y(x) = \frac{\ln x}{2} - \frac{1}{4} + \frac{1}{2x^2}}$$

b) $\underbrace{(y^3 - y^2 \operatorname{sen} x - x)}_{M}dx + \underbrace{(3xy^2 + 2y\cos x)}_{N}dy = 0$

- $\dfrac{\partial M}{\partial y} = 3y^2 - 2y \operatorname{sen} x$

  $\dfrac{\partial N}{\partial x} = 3y^2 - 2y \operatorname{sen} x$

Como $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, a eq. é exata.

$\therefore \exists\, U(x,y)$ tal que $\dfrac{\partial U}{\partial x} = y^3 - y^2 \operatorname{sen} x - x$

$\dfrac{\partial U}{\partial y} = 3xy^2 + 2y\cos x$

$\therefore U(x,y) = \int y^3 - y^2 \operatorname{sen} x - x \, dx =$

$= y^3 \cdot x + y^2 \cos x - \dfrac{x^2}{2} + G(y)$

$\dfrac{\partial U}{\partial y} = \cancel{3y^2 x} + \cancel{2y\cos x} + G'(y) = \cancel{3xy^2} + \cancel{2y\cos x} \Rightarrow G'(y) = 0 \Rightarrow G(y) = C$

$$\boxed{U(x,y) = y^3 \cdot x + y^2 \cos x - \frac{x^2}{2} + C}$$

Solução geral da eq. dif :

$$\boxed{\left\{ (x,y) : y^3 x + y^2 \cos x - \frac{x^2}{2} = C \right\}}$$

c) $\underbrace{x}_{N}\,dy - \underbrace{(xe^{y/x} + y)}_{M}\,dx = 0$

$N(x,y) = x \qquad N(kx,ky) = kx = kN(x,y)$ homog. ordem 1

$M(x,y) = -(xe^{y/x} + y) \qquad M(kx,ky) = -(kxe^{ky/kx} + ky) = -k(xe^{y/x} + y) = kM(x,y)$

homog. ordem 1.

$\therefore$ A eq. dif. é homogênea.

- $y' = \dfrac{xe^{y/x} + y}{x} = e^{y/x} + y/x$

Mudança de variaveis:

$y/x = t(x) \qquad y(x) = t(x) \cdot x$

$y' = t' \cdot x + t$

$$t'x + \cancel{t} = e^{t} + \cancel{t}$$

Variaveis Sep: $\rightarrow$ $\frac{dt}{dx} \cdot x = e^{t} \Rightarrow \frac{dt}{e^{t}} = \frac{dx}{x} \Rightarrow \int e^{-t}\,dt = \int \frac{dx}{x}$

$$\Rightarrow -e^{-t} = \ln x + C \Rightarrow e^{-t} = -\ln x + C$$

$$-t = \ln(-\ln + C) \Rightarrow t(x) = -\ln(-\ln x + C)$$

$$\therefore y/x = -\ln(-\ln x + C) \Rightarrow$$

Sol. geral

$$\boxed{y(x) = -x \cdot \ln(-\ln x + C)}$$

OBS: $-\ln x + C \geq 0$

$C \geq \ln x$

$e^{C} \geq x > 0$

d) $y = xy' - e^{y'}$

Eq. de Clairaut:

$$\cancel{y'} = \cancel{y'} + x\overbrace{y''} - e^{y'} \cdot \overbrace{y''}$$

$$0 = (x - e^{y'})\, y''$$

$\to x - e^{y'} = 0$] Sol. Sing.

$\to y'' = 0$] Sol. geral

• Sol. geral

$y'' = 0 \Rightarrow y(x) = mx + b$

Subst. na eq. dif. $y'(x) = m$

$$\cancel{mx} + b = \cancel{x \cdot m} - e^{m} \Rightarrow b = -e^{m}$$

Sol. geral

$$\boxed{y(x) = mx - e^{m}}$$

$$\begin{cases} m = 1 & y(x) = x - e \\ m = 0 & y(x) = -1 \\ m = 2 & y(x) = 2x - e^{2} \end{cases}$$

Solução singular:

$$x - e^{y'} = 0 \qquad x = e^{y'}$$

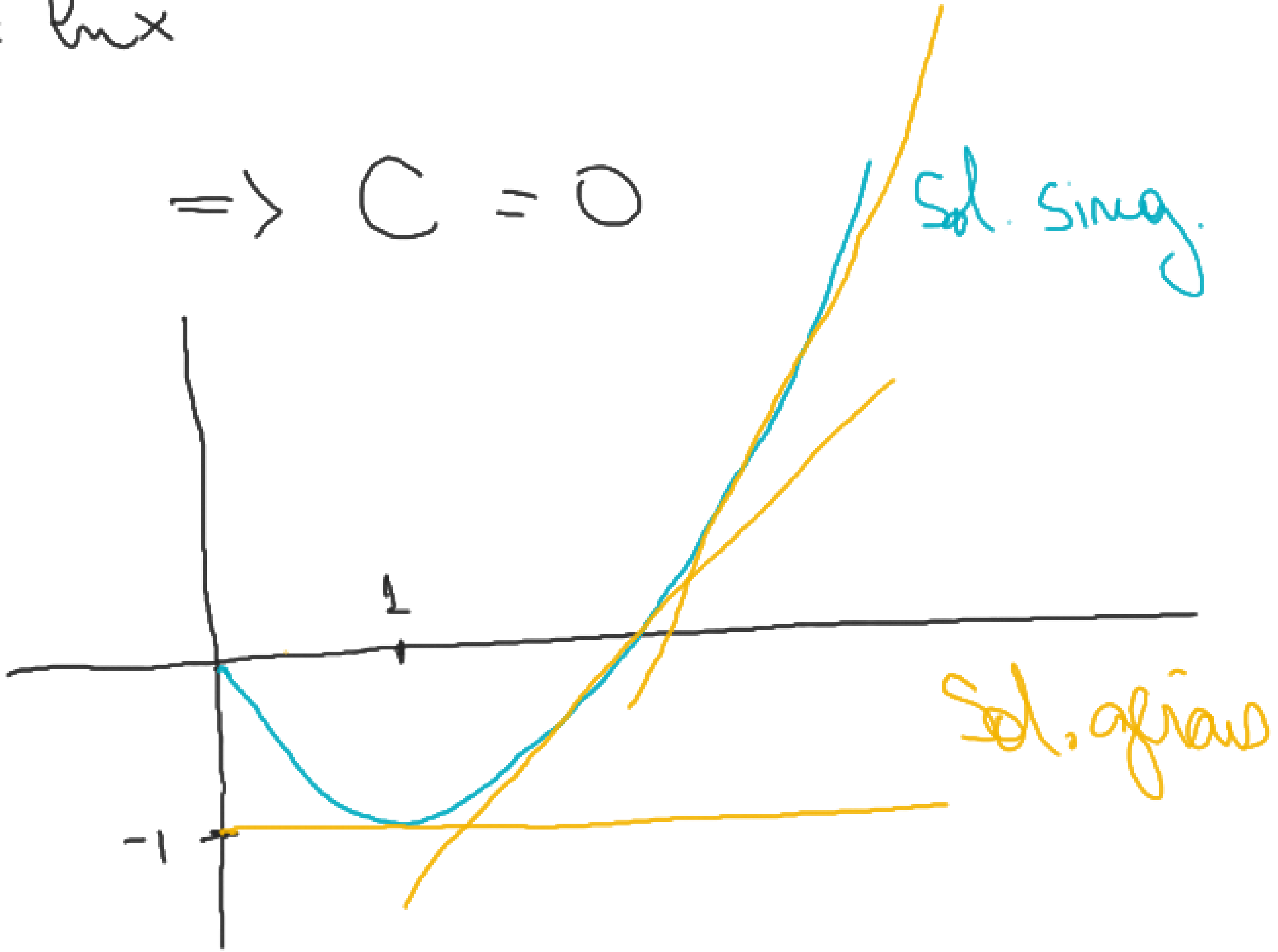
$$\ln x = y'$$

$$y(x) = \int 1 \cdot \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - \int dx = x \ln x - x + C$$

Subst. $y(x)$ na eq. dif: $y'(x) = \ln x$

$$\therefore \cancel{x \ln x} - \cancel{x} + C = \cancel{x \cdot \ln x} - \underbrace{\cancel{e^{\ln x}}}_{x} \Rightarrow C = 0$$

$$\boxed{y(x) = x \ln x - x}$$



3 _ $y' + y^2 + 3y + 2 = 0$

$y_1(x) = -1$ (sol. particular)

• Eq. Ricatti

$\boxed{\text{Sol} \quad y(x) = z(x) - 1}$

↳ Sol. de uma eq. de Bernoulli _

• Substituimos $y(x) = z(x) - 1$ na eq. dif $\rightarrow$ $y'(x) = z'$

$$z' + (z-1)^2 + 3(z-1) + 2 = 0$$

$$z' + z^2 - 2z + 1 + 3z - 3 + 2 = 0$$

$$z' + z + z^2 = 0 \Rightarrow \boxed{z' + z = -z^2}$$ Bernoulli

- Calculamos a solução $z' + z = -z^2$

$$\frac{z'}{z^2} + \frac{1}{z} = -1$$

- Mudança de variáveis: $t = 1/z(x)$ $\quad t' = -\frac{1}{z^2} \cdot z'$

$$-t' + t = -1 \implies \frac{dt}{dx} - t = 1 \implies \frac{dt}{dx} = 1 + t \implies$$

$$\frac{dt}{1+t} = dx \implies \int \frac{dt}{1+t} = \int dx \implies \ln(1+t) = x + C$$

$$\therefore 1 + t = e^{x+C} \implies t(x) = e^x \cdot e^C - 1$$

- Variáveis originais: $z(x) = \frac{1}{t(x)} = \frac{1}{C \cdot e^x - 1}$

- Solução geral: $y(x) = z(x) - 1$

$$\boxed{y(x) = \frac{1}{C e^{x} - 1} - 1}$$