Lista 5 de EDO
$2^o$ semestre de 2013
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Transformada de Laplace
Transformada de Laplace Inversa
Convolução

1. Calcule a Transformada de Laplace das seguintes funções.

   (a) $\operatorname{senh}(at) = \frac{e^{at}-e^{-at}}{2}$. *Resp:* $\frac{a}{s^2-a^2}$, $s > |a|$;

   (b) $\cosh(at) = \frac{e^{at}+e^{-at}}{2}$. *Resp:* $\frac{s}{s^2-a^2}$, $s > |a|$;

   (c) $5t - 3$. *Resp:* $\frac{5-3s}{s^2}$, $s > 0$;

   (d) $2t^2 - e^{-t}$. *Resp:* $\frac{4+4s-s^3}{s^3(s+1)}$, $s > 0$;

   (e) $6\operatorname{sen}(2t) - 5\cos(2t)$. *Resp:* $\frac{12-5s}{s^2+4}$, $s > 0$;

   (f) $(\operatorname{sen} t - \cos t)^2$. *Resp:* $\frac{s^2-2s+4}{s(s^2+4)}$, $s > 0$;

   (g) $-4\operatorname{senh}(5t) + 3\cosh(5t)$. *Resp:* $\frac{3s-20}{s^2-25}$, $s > 5$;

   (h) $4\cos^2(2t)$. *Resp:* $\frac{2}{s} + \frac{2s}{s^2+16}$, $s > 0$;

   (i) $3t^4 - 2t^3 + 4e^{-3t} - 2\operatorname{sen}(5t) + 3\cos(2t)$. *Resp:* $\frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} - \frac{10}{s^2+25} + \frac{3s}{s^2+4}$, $s > 0$;

   (j) $t^3e^{-3t}$. *Resp:* $\frac{6}{(s+3)^4}$, $s > 0$;

   (k) $2e^{3t}\operatorname{sen}(4t)$. *Resp:* $\frac{8}{s^2-6s+25}$, $s > 0$;

   (l) $(t+2)^2e^t$. *Resp:* $\frac{4s^2-4s+2}{(s-1)^3}$, $s > 0$;

   (m) $t\operatorname{sen}(at)$. *Resp:* $\frac{2as}{(s^2+a^2)^2}$;

   (n) $t\cos(at)$. *Resp:* $\frac{s^2-a^s}{(s^2+a^2)^2}$;

   (o) $t^2\operatorname{sen}(at)$. *Resp:* $\frac{6s^2a-2a^3}{(s^2+a^2)^3}$;

2. Calcule as seguintes integrais.

   (a) $\displaystyle\int_0^\infty te^{-3t}\operatorname{sen} t$. *Resp:* $\frac{3}{50}$;

   (b) $\displaystyle\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}$. *Resp:* $\ln 2$;

(c) $\displaystyle\int_0^\infty \frac{\operatorname{sen}^2 t}{t^2}$. *Resp:* $\frac{\pi}{2}$; (*Dica:* Use partes para reduzir ao cálculo de $\int_0^\infty \frac{\operatorname{sen} 2t}{t}$)

3. Determine a transformada de Laplace inversa das seguintes funções.

   (a) $\frac{3s-12}{s^2+8}$. *Resp:* $3\cos(2\sqrt{2}t) - 3\sqrt{2}\operatorname{sen}(2\sqrt{2}t)$;

   (b) $\frac{2s-5}{s^2-9}$. *Resp:* $2\cosh(3t) - \frac{5}{3}\operatorname{senh}(3t)$;

   (c) $\frac{1}{s^5}$. *Resp:* $t^4/24$;

   (d) $\frac{12}{4-3s}$. *Resp:* $-4e^{4t/3}$;

   (e) $\frac{s}{(s+1)^5}$. *Resp:* $\frac{e^{-t}}{24}(4t^3 - t^4)$;

   (f) $\frac{3s-14}{s^2-4s+8}$. *Resp:* $e^{2t}(3\cos(2t) - 4\operatorname{sen}(2t)$;

   (g) $\frac{1}{(s-a)^3}$. *Resp:* $\frac{t^2}{2}e^{at}$; (*Dica:* $\mathcal{L}^{-1}(\frac{1}{s-a}) = e^{at}$)

   (h) $\frac{s}{(s^2-a^2)^2}$. *Resp:* $\frac{t}{2}\operatorname{senh}(at)$; (*Dica:* $\mathcal{L}^{-1}(\frac{1}{s^2-a^2}) = \operatorname{senh}(at)$)

   (i) $\ln\left(\frac{s+2}{s+1}\right)$. *Resp:* $\frac{e^{-t}-e^{-2t}}{t}$;

   (j) $\frac{1}{s^3(s+1)}$. *Resp:* $1 - t + \frac{t^2}{2} - e^{-t}$;

   (k) $\frac{1}{s(s+1)^3}$. *Resp:* $1 - (1 + t + \frac{t^2}{2})e^{-t}$;

   (l) $\frac{3s+16}{s^2-s-6}$. *Resp:* $5e^{3t} - 2e^{-2t}$;

   (m) $\frac{27-12s}{(s^2+9)(s+4)}$. *Resp:* $3e^{-4t} - 3\cos(3t)$;

   (n) $\frac{s-1}{(s^2+2s+2)(s+3)}$. *Resp:* $\frac{1}{5}e^{-t}(4\cos t - 3\operatorname{sen} t) - \frac{4}{5}e^{-3t}$;

4. Calcule, usando convolução, as transformadas de Laplace inversas das seguintes funções.

   (a) $\frac{1}{(s+3)(s-1)}$. *Resp:* $\frac{1}{4}(e^t - e^{-3t})$;

   (b) $\frac{1}{(s+1)(s^2+1)}$. *Resp:* $\frac{1}{2}(\operatorname{sen} t - \cos t + e^{-t})$;

   (c) $\frac{s^2}{(s^2-a)^2}$. *Resp:* $\frac{1}{2}t\cos(2t) + \frac{1}{4}\operatorname{sen}(2t)$;

   (d) $\frac{1}{(s^2+1)^3}$. *Resp:* $\frac{1}{8}(-3t\cos t + (3 - t^2)\operatorname{sen} t)$;