Lista 6 de EDO
2^o semestre de 2013
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Resolução de EDO usando
Transformada de Laplace
Exponencial de Matrizes

1. Resolva os PVI's usando Transformada de Laplace.

   (a) $y'' - 2y' + 5y = 0$, $y(0) = 2$, $y'(0) = 4$. *Resp:* $2e^t \cos 2t + e^t \,\text{sen}2t$;

   (b) $y'' + 6y' + 9y = 0$, $y(0) = -1$, $y'(0) = 6$. *Resp:* $-e^{-3t} + 3te^{-3t}$;

   (c) $y'' + y = t^2 + 2$, $y(0) = 1$, $y'(0) = -1$. *Resp:* $t^2 + \cos t - \text{sen}t$;

   (d) $y'' - 7y' + 10y = 9\cos t + 7\,\text{sen}t$, $y(0) = 5$, $y'(0) = -4$. *Resp:* $\cos t - 4e^{5t} + 8e^{2t}$;

   (e) $\begin{array}{rcl} x' &=& 3x - 2y; \quad x(0) = 1 \\ y' &=& 3y - 2x; \quad y(0) = 1 \end{array}$. *Resp:* $x = e^t$, $y = e^t$;

   (f) $\begin{array}{rcl} x' &=& y + \text{sen}t; \quad x(0) = 2 \\ y' &=& x + 2\cos t; \quad y(0) = 0 \end{array}$. *Resp:* $\begin{array}{rcl} x &=& (7/4)(e^t + e^{-t}) - (3/2)\cos t \\ y &=& (7/4)(e^t - e^{-t}) + (1/2)\,\text{sen}t \end{array}$;

   (g) $\begin{array}{rcl} x' + y &=& 3 - t; \quad x(0) = 0 \\ x + y' &=& 0; \quad y(0) = 0 \end{array}$.

   Resp: $\begin{array}{rcl} x &=& (e^t - e^{-t})/2 - (1/2)[e^{t-2} - e^{-(t-2)}](t-2) \\ y &=& 1 - (e^t + e^{-t})/2 - [1 - (e^{t-2} + e^{-(t-2)})/2](t-2) \end{array}$;

2. Encontre a solução geral do sistema $X'(t) = A \cdot X(t)$.

   (a) $\begin{bmatrix} -1 & 3/4 \\ -5 & -3 \end{bmatrix}$; *Resp:* $c_1 e^{3t/2} \begin{bmatrix} 3 \\ 10 \end{bmatrix} + c_2 e^{t/2} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$; *Resp:* $c_1 e^{-t} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + c_3 e^{5t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$;

   (c) $\begin{bmatrix} -1 & 1 \\ 8 & 1 \end{bmatrix}$; *Resp:* $c_1 e^{3t} \begin{bmatrix} 1 \\ 4 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}$; *Resp:* $c_1 \begin{bmatrix} 2\cos 2t \\ \cos 2t + \text{sen}2t \end{bmatrix} + c_2 \begin{bmatrix} 2\,\text{sen}2t \\ \text{sen}2t - \cos 2t \end{bmatrix}$

   (e) $\begin{bmatrix} -1 & -2 \\ 8 & -1 \end{bmatrix}$; *Resp:* $c_1 e^{-t} \begin{bmatrix} \cos 4t \\ 2\,\text{sen}4t \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} \text{sen}4t \\ -2\cos 4t \end{bmatrix}$

(f) $\begin{bmatrix} 3 & -2 \\ 0 & 3 \end{bmatrix}$; *Resp:* $c_1e^{3t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2e^{3t}\begin{bmatrix} -2t \\ 1 \end{bmatrix}$

3. Determine a solução do PVI.

(a) $X'(t) = \begin{bmatrix} 3 & -2 \\ 0 & 3 \end{bmatrix} \cdot X(t)$, com $X(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$; *Resp:* $\begin{bmatrix} 2e^{4t} + e^{-2t} \\ 2e^{4t} - e^{-2t} \end{bmatrix}$

(b) $X'(t) = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix} \cdot X(t)$, com $X(\pi) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; *Resp:* $\begin{bmatrix} -e^{-2(t-\pi)}\cos t \\ -e^{-2(t-\pi)}(\cos t - \text{sen} t) \end{bmatrix}$

(c) $X'(t) = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} \cdot X(t)$, com $X(0) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$; *Resp:* $\begin{bmatrix} -2e^{3t}(1-t) - 5e^{3t}t \\ -2e^{3t}t + 5e^{3t}(1+t) \end{bmatrix}$