

4ª Lista , EDO

Professor: Sergio Licanic

$t \geq 0$	$\mathcal{L}(f)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\text{sen}(at)$	$\frac{a}{s^2+a^2}$
$\text{cos}(at)$	$\frac{s}{s^2+a^2}$
$\text{senh}(at)$	$\frac{a}{s^2-a^2}$
$\text{cosh}(at)$	$\frac{s}{s^2-a^2}$

Para resolver esta lista lembrar _____

Primeiro Teorema de Translação:

$$\mathcal{L}(e^{at}f(t)) = \mathcal{L}(f)(s-a).$$

Segundo Teorema de Translação:

$$\mathcal{L}(\mathcal{U}(t-a)f(t-a)) = e^{-as}\mathcal{L}(f).$$

Teorema:

$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{(n-1)}f(0) - \dots - f^{(n-1)}(0)$$

Teorema:

$$\mathcal{L}(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} \mathcal{L}(f)$$

Teorema:

$$\mathcal{L}(f * g) = \mathcal{L}(f) \cdot \mathcal{L}(g)$$

1_ Calcule:

a) $\mathcal{L}\{t^{10} + 3t^4\}$ b) $\mathcal{L}\{\text{sen}(7t)\}$ c) $\mathcal{L}\{t^3 \text{senh}(2t)\}$ d) $\mathcal{L}\{t^4 e^{4t}\}$

e) $\mathcal{L}\{e^{3t} \cosh 5t\}$ f) $\mathcal{L}\{e^{2t} \text{senh}(10t)\}$ g) $\mathcal{L}\left\{\frac{e^{at} - e^{bt}}{a-b}\right\}$

h) $\mathcal{L}\{e^{3t-2}\mathcal{U}(t-1)\}$ i) $\mathcal{L}\{\mathcal{U}(t-\pi)\text{sen}(10t-2)\}$ j) $\mathcal{L}\{e^{4t}\mathcal{U}(t-\pi)\}$

2_ Resolva:

- a) $\mathcal{L}^{-1}\left\{\frac{2}{(s-2)^3}\right\}$, b) $\mathcal{L}^{-1}\left\{(1+e^{-2s})/(s+1)\right\}$, c) $\mathcal{L}^{-1}\{e^{2s}(s^3-1)\}$
d) $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2)^2}\right\}$, e) $\mathcal{L}^{-1}\{s^2/(s^2-1)^2\}$ f) $\mathcal{L}^{-1}\{e^{-6s}(s-1)^3\}$
g) $\mathcal{L}^{-1}\{s/(s-1)^3(s^2+3)\}$, h) $\mathcal{L}^{-1}\{e^{3s}(s-1)^3\}$ i) $\mathcal{L}^{-1}\{4/(s-3)^4\}$,
j) $\mathcal{L}^{-1}\{1/(s^2+6s+34)(s-1)^2\}$, k) $\mathcal{L}^{-1}\{s^2-1/(s-3)^4\}$, l) $\mathcal{L}^{-1}\{(s-1)^2/(s^2+6s)\}$,
m) $\mathcal{L}^{-1}\{e^{\pi s}/(s^2+2s-5)\}$, n) $\mathcal{L}^{-1}\{e^{3(s-8)}(s^2-1)\}$ o) $\mathcal{L}^{-1}\{s/[(s-3)^2+1]\}$,
p) $\mathcal{L}^{-1}\{s^2/(s^2+s-7)\}$, q) $\mathcal{L}^{-1}\{(e^{2\sqrt[4]{3}}s-1)/(s-3)^4\}$, r) $\mathcal{L}^{-1}\{(s-1)/(s^2+s+1)\}$

3. Encontre a transformada de Laplace das seguintes funções (Usar, $\mathcal{U}(t-a)$, a função degrau unitário):

- a) $f(t) = \begin{cases} -1, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$, b) $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$
c) $f(t) = \begin{cases} 0, & 0 \leq t < \pi/2 \\ \cos t, & t \geq \pi/2 \end{cases}$, d) $f(t) = \begin{cases} \operatorname{sen} t, & 0 \leq t < \pi \\ 1 + \cos t, & t \geq \pi \end{cases}$
e) $f(t) = \begin{cases} 2, & 0 \leq t < 1 \\ e^{3t}t^4, & 1 \leq t < 2 \\ 2, & t \geq 2 \end{cases}$ f) $f(t) = \begin{cases} t^2 \operatorname{sen}(t), & 0 \leq t < \pi \\ e^{2t} \cos 4t + t^2 \operatorname{sen} t, & \pi \leq t < 2\pi \\ t^2 \operatorname{sen}(t), & t \geq 2\pi \end{cases}$

4. Use a transformada de Laplace para resolver,

- a) $y'' - y' - e^t \operatorname{sen}(2t) = 0$, $y(0) = 2$, $y'(0) = 1$ b) $y'' + y' + y = e^{3t}$, $y(0) = 1$, $y'(0) = 2$
c) $y'' - 6y' + 9y = te^{3t}$, $y(0) = 1$, $y'(0) = 1$ d) $y'' + 4y' + 6y = 1 - e^{-t}$, $y(0) = 1$, $y'(0) = 1$,
e) $y' - y = e^t \operatorname{sen}(2t)$, $y(0) = 0$ f) $y' - y = e^{5t} \operatorname{sen}(3t)$, $y(0) = 4$
g) $2y'' - y' = t^2 + e^{2t} \operatorname{sen}(2t)$, $y(0) = 5$, $y'(0) = 1$ h) $y'' + 2y' + 2y = e^{3t}(1+t)^4$, $y(0) = 1$, $y'(0) = 1$
i) $y'' - y' + 2y = t$, $y(0) = 0$, $y'(0) = 1$, j) $y'' + 4y' + 6y = 10e^{2t}$, $y(0) = 1$, $y'(0) = 1$
k) $y''(t) = 1 - \operatorname{sen} t - \int_0^t y(\tau) d\tau$ l) $\frac{dy}{dx} + 6y(t) + 9 \int_0^t y(\tau) d\tau = 1$
m) $\begin{cases} y^{(4)} - y = e^t \cos(6t) \\ y(0) = y'(0) = 0 \\ y''(0) = y'''(0) = 1 \end{cases}$ n) $\begin{cases} 2y'' - 5y' + y = f(t) \\ y(0) = 1, \\ y'(0) = 2. \end{cases}$ $f(t)$ como em ex.3

5. Use a transformada de Laplace para encontrar f ,

- a) $f(t) + \int_0^t (t-\tau)f(\tau) d\tau = t$, b) $f(t) = 2t - 4 \int_0^t \operatorname{sen} \tau f(t-\tau) d\tau$.