Universidade Federal Fluminense
EGM - Instituto de Matemática
GMA - Departamento de Matemática Aplicada

LISTA 3 - 2010-2

Integração por partes
Integrais de potências de funções trigonométricas

Nos exercícios 1 a 18, calcule a integral indicada, utilizando a técnica de integração por partes.

1. $\int \operatorname{arcsen} x \, dx$
2. $\int x \operatorname{sen} x \, dx$
3. $\int x^2 \ln x \, dx$
4. $\int x^2 \cos x \, dx$
5. $\int x \arctan x \, dx$
6. $\int \sec^3 x \, dx$
7. $\int (\ln x)^3 \, dx$
8. $\int \sqrt{x} \, \ln x \, dx$
9. $\int x(\ln x)^2 \, dx$
10. $\int x \, 3^x \, dx$
11. $\int x \sec^2 x dx$
12. $\int \operatorname{sen}(\ln x) \, dx$
13. $\int \operatorname{arcsec} \sqrt{x} \, dx$
14. $\int \dfrac{\ln(\ln x)}{x} \, dx$
15. $\int \dfrac{\arctan \sqrt{x}}{\sqrt{x}} \, dx$
16. $\int \tan^2 x \sec^3 x \, dx$
17. $\int \csc^5 x \, dx$
18. $\int \operatorname{sen} 3x \cos 2x \, dx$

Nos exercícios 19 a 30 calcule a integral do produto ou quociente de potências de funções trigonométricas.

19. $\int \operatorname{sen}^4 x \, dx$
20. $\int \operatorname{sen}^4 x \cos^2 x \, dx$
21. $\int \operatorname{sen}^3 x \cos^2 x \, dx$
22. $\int \cos^5 x \, dx$
23. $\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \, \operatorname{sen}^3 x \, dx$
24. $\int \dfrac{\operatorname{sen}^3 x}{\cos^4 x} \, dx$
25. $\int_0^{\frac{1}{2}} \cos(\pi x) \cos\left(\dfrac{\pi x}{2}\right) \, dx$
26. $\int \tan^2 x \sec^3 x \, dx$
27. $\int \tan^5 x \sec^3 x \, dx$
28. $\int \tan^3 x \sqrt{\sec x} \, dx$
29. $\int \dfrac{\tan^5 x}{\sec^3 x} \, dx$
30. $\int \cot^4 x \, dx$

Lista de algumas fórmulas exponenciais e trigonométricas que eventualmente serão usadas nas resoluções de algumas integrais:

(I) $a^x = e^{x \ln a}, \;\; a \in \mathbb{R}, a > 0$

(II) $\operatorname{sen}^2 x + \cos^2 x = 1$ (III) $1 + \tan^2 x = \sec^2 x$ (IV) $1 + \cot^2 x = \csc^2 x$

(V) $\cos^2 x = \dfrac{1 + \cos 2x}{2}$ (VI) $\operatorname{sen}^2 x = \dfrac{1 - \cos 2x}{2}$

(VII) $\operatorname{sen} a \cos b = \dfrac{\operatorname{sen}(a-b) + \operatorname{sen}(a+b)}{2}$ (VIII) $\operatorname{sen} a \operatorname{sen} b = \dfrac{\cos(a-b) - \cos(a+b)}{2}$

(IX) $\cos a \cos b = \dfrac{\cos(a-b) + \cos(a+b)}{2}$

RESPOSTAS DA LISTA 3

1. $x \operatorname{arcsen} x + \sqrt{1-x^2} + C$
2. $\operatorname{sen} x - x \cos x + C$
3. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$
4. $x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C$
5. $\frac{1}{2}\left(x^2 \arctan x - x + \arctan x\right) + C$
6. $\frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$
7. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$
8. $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{4}{9}x^{\frac{3}{2}} + C$
9. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2(\ln x) + \frac{1}{4}x^2 + C$
10. $\frac{1}{\ln 3}\ x\ 3^x - \frac{1}{(\ln 3)^2}\ 3^x + C$
11. $x \tan x - \ln|\sec x| + C$
12. $\frac{1}{2}x\left(\operatorname{sen}(\ln x) - \cos(\ln x)\right) + C$
13. $x \operatorname{arcsec}\sqrt{x} - (x-1)^{\frac{1}{2}} + C$
14. $\ln x \ln(\ln x) - \ln x + C$
15. $2\sqrt{x}\ \arctan(\sqrt{x}) - \ln(1+x) + C$
16. $\frac{1}{4}\sec^3 x \tan x - \frac{1}{8}\sec x \tan x - \frac{1}{8}\ln|\sec x + \tan x| + C$
17. $-\frac{1}{4}\csc^3 x \cot x - \frac{3}{8}\cot x \csc x - \frac{3}{8}\ln|\csc x - \cot x| + C$
18. $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$
19. $\frac{3}{8}x + \frac{1}{32}\operatorname{sen} 4x - \frac{1}{4}\operatorname{sen} 2x + C$
20. $\frac{1}{16}x - \frac{1}{64}\operatorname{sen} 4x - \frac{1}{48}\operatorname{sen}^3\ 2x + C$
21. $-\frac{1}{5}\cos^3 x \operatorname{sen}^2 x - \frac{2}{15}\cos^3 x + C$
22. $\frac{1}{5}\operatorname{sen}^5 x - \frac{2}{3}\operatorname{sen}^3 x + \operatorname{sen} x + C$
23. $\frac{2}{7}\cos^{\frac{7}{2}} x - \frac{2}{3}\cos^{\frac{3}{2}} x\Big]_0^{\frac{\pi}{2}} = \frac{8}{21}$
24. $\frac{1}{3}\cos^{-3} x - \cos^{-1} x + C$
25. $\left(\frac{1}{3\pi}\operatorname{sen}\frac{3\pi x}{2} + \frac{1}{\pi}\operatorname{sen}\frac{\pi x}{2}\right)\Big]_0^{\frac{1}{2}} = \frac{2\sqrt{2}}{3\pi}$
26. $\frac{1}{4}\sec^3 x \tan x - \frac{1}{8}\sec x \tan x - \frac{1}{8}\ln|\sec x + \tan x| + C$
27. $\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$
28. $\frac{2}{5}\sec^{\frac{5}{2}} x - 2\sec^{\frac{1}{2}} x + C$
29. $\sec x + 2\sec^{-1} x - \frac{1}{3}\sec^{-3} x + C$
30. $\cot x - \frac{1}{3}\cot^3 x + x + C$