

UFF 2018

Properties of Certain Graph Centrality Measures in Tournaments

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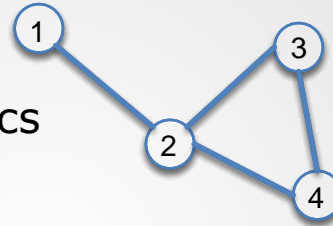
João Carlos Soares de Mello



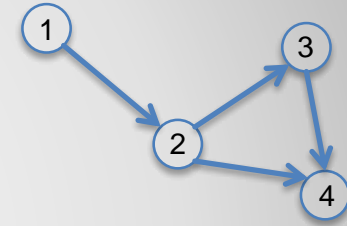
Theoretical Background

- Graph Theory

- $G(V,E)$ – vertices and edges/arcs
- Directed or Undirected
- Weighted or Unweighted
- Neighbours
- Adjacency Matrix



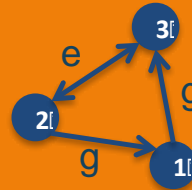
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$



$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

- ❖ Tournament Ranking

- ❖ Vertices: teams/players
- ❖ Arcs: Victories (g) and ties (e)



Theoretical Background

- Centrality – importance of a vertex

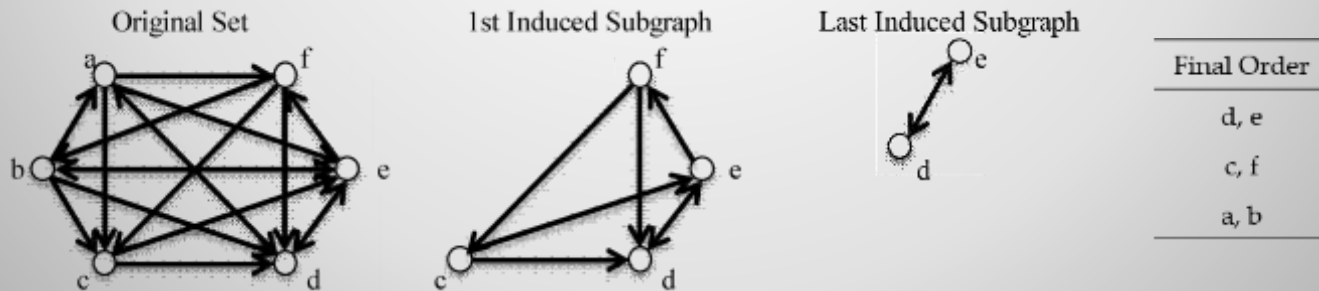
- Degree $C_D(v_i) = d(v_i) = \sum_{j=1}^n a_{ij}$

- Eigenvector $Ax = \lambda x$

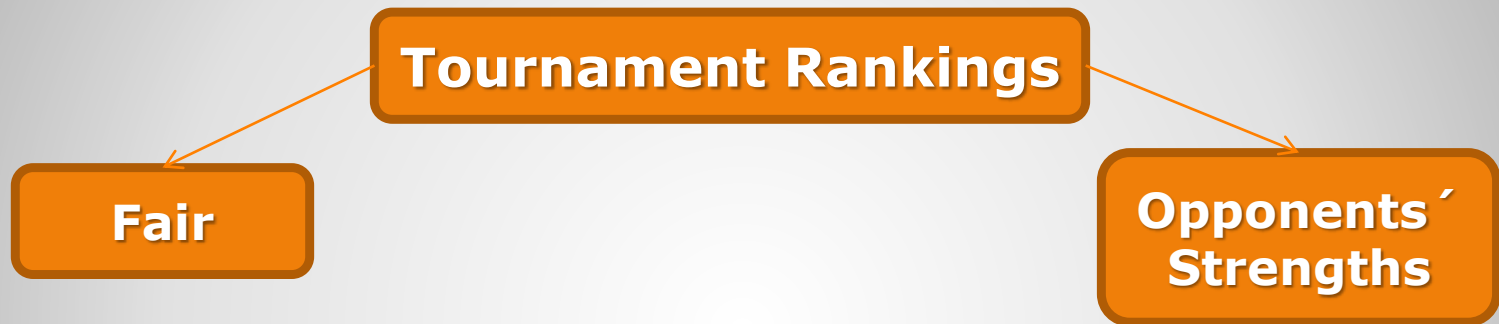
- Considers neighbours' centralities → considers the opponents' strengths

- Layer

- Ranks from bottom to top, based on degree of each subgraph



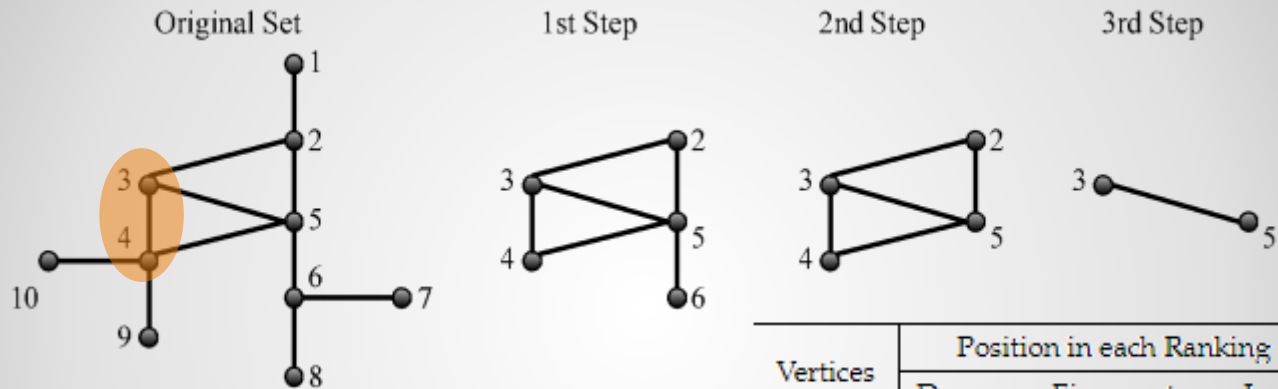
Properties of Centrality Measures



Properties of Centrality Measures



Properties of Centrality Measures



Final Order
3,5
2,4
6
1,7,8,9,10

Vertices	Position in each Ranking		
	Degree	Eigenvector	Layer
1	6	8	6
2	3	4	3
3	3	2	1
4	1	3	3
5	1	1	1
6	3	5	5
7	6	9	6
8	6	9	6
9	6	6	6
10	6	6	6

✓ Considers opponents' strengths

Properties of Centrality Measures

Tournament Rankings

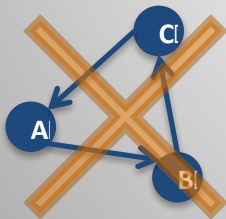
Fair

Opponents' Strengths

Arrow's axioms

- Transitivity
- Independence of irrelevant alternatives
- Universality

Coherent Tournaments



GROUP A

	W	D	L	PTS
Uruguay	3	0	0	9
Russia	2	0	1	6
Saudi Arabia	1	0	2	3
Egypt	0	0	3	0

GROUP G

	W	D	L	PTS
Belgium	3	0	0	9
England	2	0	1	6
Tunisia	1	0	2	3
Panama	0	0	3	0

Properties of Centrality Measures

- **Proposition 1:** *The layer and the degree centralities provide the same ranking in coherent tournaments, whereas the eigenvector centrality only provides the same ranking in certain cases.*
 - **No ties:** *The eigenvector centrality for the player that won every other player is $\mathbf{1}$; for every other player it is $\mathbf{0}$*
 - ...
 - **1 tie between the last players:** *All centralities provide the same ranking*

	P	W	D	L	F	-	A	Pts	Eigen. Points
1. Uruguay	3	3	0	0	5	-	0	9	1
2. Russia	3	2	0	1	8	-	4	6	0
3. Saudi Arabia	3	1	0	2	2	-	7	3	0
4. Egypt	3	0	0	3	2	-	6	0	0

Eigen. Ranking	
1	Uruguay
2	Russia, Saudi Arabia, Egypt

WorldCup 2018, Group A

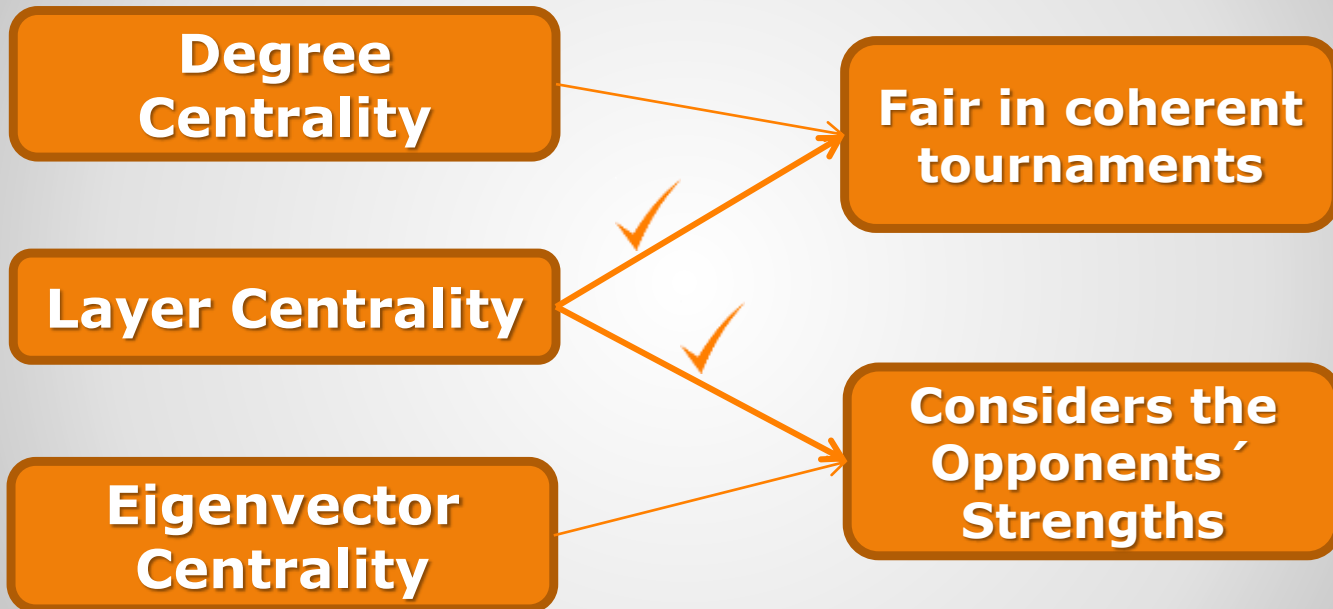
$$\begin{bmatrix}
 0 & g & g & \dots & g & g \\
 0 & 0 & g & \dots & g & g \\
 0 & 0 & 0 & \dots & g & g \\
 \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & 0 & \dots & 0 & g \\
 0 & 0 & 0 & \dots & 0 & 0
 \end{bmatrix}$$

Properties of Centrality Measures

- **Proposition 2:** *For coherent tournaments, the layer centrality ranking is fair, according to Arrow's axioms*



Properties of Centrality Measures



Centralities in the 2018 World Cup (Group Stage)

- Degree (official points) = Layer Ranking

GROUP A		
	Uruguay	9
	Russia	6
	Saudi Arabia	3
	Egypt	0

GROUP B		
	Spain	5
	Portugal	5
	IR Iran	4
	Morocco	1

GROUP D		
	Croatia	9
	Argentina	4
	Nigeria	3
	Iceland	1

GROUP G		
	Belgium	9
	England	6
	Tunisia	3
	Panama	0

Eigen. Ranking

- | | |
|---|-----------------------------|
| 1 | Uruguay |
| 2 | Russia, Saudi Arabia, Egypt |

**Coherent tournament
(no ties)**

Eigen. Ranking

- | | |
|---|----------|
| 1 | Spain |
| 2 | Portugal |
| 3 | Iran |
| 4 | Morocco |

Rankings:

Degree = Layer
= Eigenvector


Eigen. Ranking

- | | |
|---|--------------------------|
| 1 | Belgium |
| 2 | England, Tunisia, Panama |

**Coherent tournament
(no ties)**

Centralities in the 2018 World Cup (Group Stage)

- Degree (official points) \neq Layer Ranking

GROUP C		
 France	7	
 Denmark	5	
 Peru	3	
 Australia	1	

France 	FULL-TIME	2-1	 Australia
Peru 	FULL-TIME	0-1	 Denmark
Denmark 	FULL-TIME	1-1	 Australia
France 	FULL-TIME	1-0	 Peru
Australia 	FULL-TIME	0-2	 Peru
Denmark 	FULL-TIME	0-0	 France

	France	Denmark	Peru	Australia
France	0	e	g	g
Denmark	e	0	g	e
Peru	0	0	0	g
Australia	0	e	0	0

Layer Ranking

- | | |
|---|-----------------|
| 1 | France, Denmark |
| 3 | Peru |
| 4 | Australia |

Centralities in the 2018 World Cup (Group Stage)

- Degree (official points) \neq Layer Ranking

GRUPE		
	Brazil	7
	Switzerland	5
	Serbia	3
	Costa Rica	1

Costa Rica 	FULL-TIME	0-1	 Serbia
Brazil 	FULL-TIME	1-1	 Switzerland
Brazil 	FULL-TIME	2-0	 Costa Rica
Serbia 	FULL-TIME	1-2	 Switzerland
Serbia 	FULL-TIME	0-2	 Brazil
Switzerland 	FULL-TIME	2-2	 Costa Rica

	Brazil	Switzerland	Serbia	Costa Rica
Brazil	0	e	g	g
Switzerland	e	0	g	e
Serbia	0	0	0	g
Costa Rica	0	e	0	0

Layer Ranking

- 1 | Brazil, Switzerland
- 3 | Serbia
- 4 | Costa Rica

Centralities in the 2018 World Cup (Group Stage)

- Degree (official points) \neq Layer Ranking

GROUP F		
	Sweden	6
	Mexico	6
	Korea Republic	3
	Germany	3

Germany 	FULL-TIME	0-1	 Mexico
Sweden 	FULL-TIME	1-0	 Korea Republic
Korea Republic 	FULL-TIME	1-2	 Mexico
Germany 	FULL-TIME	2-1	 Sweden
Korea Republic 	FULL-TIME	2-0	 Germany
Mexico 	FULL-TIME	0-3	 Sweden

	Sweden	Mexico	Korea Rep.	Germany
Sweden	0	g	g	0
Mexico	0	0	g	g
Korea Rep.	0	0	0	g
Germany	g	0	0	0

Layer Ranking

- Sweden
- Mexico
- Korea Rep., Germany

Eigenvector Ranking

- Sweden
- Mexico
- Germany
- Korea Republic

Centralities in the 2018 World Cup (Group Stage)

- Degree (official points) \neq Layer Ranking

GROUP H		
	Colombia	6
	Japan	4
	Senegal	4
	Poland	3

Colombia 	FULL-TIME	1-2		Japan
Poland 	FULL-TIME	1-2		Senegal
Japan 	FULL-TIME	2-2		Senegal
Poland 	FULL-TIME	0-3		Colombia
Japan 	FULL-TIME	0-1		Poland
Senegal 	FULL-TIME	0-1		Colombia

	Colombia	Japan	Senegal	Poland
Colombia	0	0	g	g
Japan	g	0	e	0
Senegal	0	e	0	g
Poland	0	g	0	0

Layer Ranking

- Japan
- Colombia
- Senegal
- Poland

Eigen. Ranking

- Colombia
- Japan
- Senegal
- Poland

Layer Centrality in Tournaments

- Considers the matches between teams left in the subgraph
 - Negative outliers are disregarded
 - Positive outliers are considered
 - Match between 1st and 2nd decides
 - Less aggressive knockout
 - 1 tournament – different strengths

Germany  FULL-TIME 2-1  Sweden

	Sweden	Mexico	Korea Rep.	Germany
Sweden	0	g	g	0
Mexico	0	0	g	g
Korea Rep.	0	0	0	g
Germany	g	0	0	0

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Thank you!

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