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Condorcet Method with Weakly Rational Decision Makers: A Case Study in the Formula 1 Constructors' Championship

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Abstract

The ordinal methods are widely used to establish rankings in sports competitions because sporting results are on an ordinal scale. However, these methods assume that the decision maker is highly rational and provide a full ranking. This paper analyses the case in which the decision maker is weakly rational. In this case, the decision maker respects the transitivity of preferences but not indifference. It also shows how to adapt the Condorcet method to deal with this situation, using the results of the 2013 Formula 1 Constructors' World Championship.

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Keywords: Condorcet Method; Weak racionality; Formula 1; Transitivity; Sports evaluation.

1. Introduction

A tournament is composed of a set of various games, or rounds, the results of which are aggregated to establish the final result of the competition, according to the description given by [1]. In some cases there is a complete aggregation, in others, each result indicates the next games to be held. In either of the two cases, if each game or round is interpreted as a criterion, or a decision maker, the final result of the championship is a multi-criteria problem, normally ordinal, and the decision-maker is considered to be strongly rational.

Various examples of the use of OR to evaluate competitions can be cited. As an example in Formula 1 we can cite: the work by [2], who discusses methods to establish rankings and uses as an example the results of the 1998 Formula 1 drivers championship; and by [3], who used multi-criteria methods to establish the ranking of

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the drivers in the 2002 championship. We can also cite the use of Data Envelopment Analysis by [4] with the purpose of evaluating the efficiency of the teams which participate in the Formula 1 World Championship.

In all of these cases the decision makers (in this case, the Formula 1 races) are considered strongly rational, a basic requirement when applying basic ordinal methods [5].

The aim of this research is to study a case in which one cannot assume strongly rational decisions on the part of the decision maker. It is worthwhile highlighting at this point that the difference between a strongly rational decision maker and a weakly rational one is that while both respect transitivity for the preference relation, the weakly rational decision maker does not have the obligation of respecting the property of transitivity in relation to indifference.

This article presents a variation of the Condorcet method with weakly rational decision makers. One of the situations in which this occurs is when each alternative is evaluated not by itself, but by some components called sub-alternatives. In order to illustrate this situation, a comparison will be made of the teams which participated in the 2013 Formula 1 Constructors' World Championship.

2. Multidecision Methods

Multi-criteria Decision Support emerged strongly as a branch of Operations Research in the 1970s. However, some elementary methods had already been in existence since the French revolution, consisting of a set of methods and techniques to help in making decisions when faced with a multiplicity of criteria [6].

The manner of describing the preference structures of the decision maker varies according to the multi-criteria analysis method chosen. Ordinal methods are considered to be extremely intuitive and not very demanding both in computational terms and in relation to the necessary information on the part of the decision maker. All that is needed from the decision maker are the pre-rankings relative to each criterion [7]. To use the ordinal methods, the decision maker must rank the alternatives in accordance with the preferences or, on occasion, use a natural order such as, for example, income obtained.

While there are various ordinal methods, including some very recent ones, such as [8] and [9], the three most referenced ordinal multi-criteria methods in the literature on the subject are those of Borda, Condorcet and Copeland, plus the more elaborated variants of the basic methods. The great advantage of the facility of the use and comprehension of these methods is highlighted by [10], who applied them to a problem regarding forestry management. In [11] warns of the danger of extracting more information than one should from results which combine ordinal and cardinal information. The paper [12] uses a sequence of the ordinal methods to establish an Olympic ranking. Other applications can be seen in [13] and [14].

According to [15], no fair choice exists, in other words, there does not exist a fair multi-criteria or multi-decision maker method. A selection method is considered fair when it obeys the axioms of universality, unanimity, independence in relation to irrelevant alternatives, transitivity and totality. The Arrow theorem guarantees that, with the exception of dictator methods, no selection method attends these axioms simultaneously.

Of special interest in this study are the axioms of independence in relation to irrelevant alternatives, of transitivity and of universality. The first states that the order of preference between two alternatives must not depend on their preferences in relation to a third alternative. The transitivity axiom states that if an alternative is preferable to a second, and this is preferable to a third, then the first must be preferable to the third. Meanwhile the universality axiom requires the method to function, respecting all other axioms, for any set of preferences of the decision makers. In this way, a method which respects the axioms in some particular cases does not respect universality [16].

The Borda method, considered the precursor of the American multi-criteria school, which in essence is a rank-sum, has the great advantage of simplicity and, as a result of this, some of its variants are used in sporting competitions as described by [3] and [2]. Nevertheless, in spite of its simplicity and the ample use of its variants, the Borda method does not respect one of the most important of the Arrow axioms, that of independence in relation to irrelevant alternatives. This fact can generate distortions, in particular the extreme dependence of the results in terms of the evaluation set chosen and the possibility of dishonest manipulations.

Meanwhile, the Condorcet method, considered the precursor of the current French multi-criteria school, works with relations of outranking. The alternatives are always compared pairwise and a graph is constructed which expresses the relationship between them. This less simple method has the advantage of avoiding distortions by making the relative position of two alternatives independent of their positions relative to any other. However, it can lead to that which is called the Condorcet paradox, or intransitivity situation. This happens when alternative A outranks alternative B, which outranks C, which, in its turn, outranks alternative A. When this situation occurs it makes it impossible to generate a ranking of the alternatives.

Another basic method used in sports is the Lexicographic method, principally employed in drawing up the Olympic Games medal table [17].

2.1. The Condorcet Method

The Condorcet method requires each decision maker to rank all of the alternatives according to their preferences. However, instead of attributing a score to each alternative as in the Borda method, the method establishes relationships of outranking. Then it must be checked which alternative out of each pair of alternatives is preferred by the majority of the decision makers. In this case, we can say that this alternative is preferable in relation to the other. Graphs can be drawn representing these preference relations, in which the arc belongs to the graph if, and only if, the number of decision makers who preferred u to v is greater or equal to those who preferred v to u. These results are analogous to those obtained with the ELECTRE I method [18], provided that there is no veto nor any discordances, or indifference thresholds.

The representation of the preference relations by means of a graph greatly facilitates determining the dominant and dominated alternatives (when they exist). When only one dominant alternative exists, that is the one chosen. The Condorcet method, which is considered more fair than the Borda method, has the great disadvantage of leading to situations of intransitivity, leading to the well-known “Condorcet paradox”. This occurs when A is preferable to B, (A P B), B is preferable C (B P C) and C is preferable A (C P A), a situation known as the “Condorcet Triplet”. This means that the Condorcet method does not always lead to a pre-ranking in the set of alternatives. However, there are situations in which cycles of intransitivity do not occur. In these situations, the Condorcet method must be preferred to the Borda method [19].

When the alternatives present sub-alternatives, the evaluation is made through the components of the alternative. Thus, if a group of people is an alternative, the evaluation of the alternative is performed by the evaluation of each of the individuals in the group, who together form the sub-alternatives. The existence of relations of preference and indifference between alternatives are formalized only considering, for simplicity’s sake, the existence of two sub-alternatives for each alternative. Let A and B be two alternatives, each with sub-alternatives A1 and A2 and B1 and B2, respectively, such that A1 P A2 and B1 P B2.

Definition 1: A P B, if and only if A1 P B1 and A2 P B2.

Definition 2: A I B, if and only if A1 I B1 and A2 I B2, or if A1 P B1 and B2 P A2, or if B1 P A1 and A2 P B2; where I represents the indifference relation.

It is easy to see that the relation A P B is transitive. However, this does not occur in the indifference relation.

To illustrate this situation, there is the example with three alternatives (A, B and C), with two sub-alternatives each. For a particular decision maker, the ranking occurred as presented in Table 1.

Table 1. Ranking of the sub-alternatives.

Position	Sub-alternative
1	A1
2	B1
3	B2
4	C1
5	C2
6	A2

In this table it can be seen that A1 P B1, and B2 P A2. By Definition 2 it must be that A I B. Similarly comparing A and C we can see that A I C. However, B1 P C1 and B2 P C2, which by Definition 1 indicates that B P C, in other words, ~(B I C), which demonstrates that the indifference was not transitive. This means that, considering the evaluation of the alternatives by means of the sub-alternatives, the decision maker was weakly rational.

As can be seen as follows, this is a situation which occurs in championships in which the ranking of the teams is based on the ranking of the individuals who belong to those teams. An example of this situation is the Formula 1 World Constructors' Championship.

3. The Formula 1 World Championship

The Formula 1 World Championship began in 1950 at Silverstone, in England. This first championship was composed of 6 Grand Prix events to be held in Europe: England, Monaco, Switzerland, Belgium, France and Italy, to which would be added the result of the Indianapolis 500, in this way making it a "world" championship (in spite of the fact that the cars, teams and drivers who competed in the USA were completely different to those in Europe).

The regulations of the Formula 1 Drivers' World Championship determine that the champion driver of the season is the driver who achieves the highest number of points at the end of all the season's races. The classification of the other drivers in the championship is determined by the total number of points achieved. In the first championship the 3 worst results of the 7 races held would be discarded. The scoring was as follows: 8 points for first place; 6 for second place; 4 for third; 3 for fourth; 2 for fifth place and one point for the driver who registered the fastest lap in the race.

The scoring system underwent alterations over time, according to [20]. Table 2 presents the scoring systems used during the championships.

Table 2. Formula 1 Scoring Systems

Time./Pos.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1950-1959	8	6	4	3	2					
1960	8	6	4	3	2	1				
1961-1990	9	6	4	3	2	1				
1991-2002	10	6	4	3	2	1				
2003-2009	10	8	6	5	4	3	2	1		
2010-2013	25	18	15	12	10	8	6	4	2	1

As can be seen in Table 2, from 2010 onwards, a larger number of drivers came to score in each race. However, in each race, only the first ten drivers received points, according to the scoring for each place as shown in Table 2. A total of 19 races were held during the 2013 championship, with 23 drivers competing in each race, 2 drivers per team, making 11 teams participating in the constructors' championship. It is important to note that there were 23 drivers, and not 22 as expected, because in race number 18 (the American Grand Prix), the Lotus Renault team replaced their driver Kimi Raikkonen by Heikki Kovalainen. Because of the substitution, both drivers count as one alternative, in other words, as if they were only one driver.

The regulation cited above in Table 2 is, in fact, a variation of the Borda method. The most obvious difference in relation to the traditional Borda method is that the first placed drivers score more points while in the original method they score fewer points. This is justified by the fact that not all competitors finish, or even participate, in all of the races. A non-participating driver would not score, which would be a better situation than the first placed driver. It is therefore an alteration which permits an improvement in the operationalization of the method, without bringing any damaging consequences.

The regulations also preview the possibility of ties at the end of the final scoring, establishing successive tie-breaking criteria. In this way, the regulations in fact use the Lexicographic method, with the most important criterion (and therefore the first to be used) being the scoring obtained with the modified Borda method. When two or more alternatives have the same number of points at the end of the championship then the greater number of victories of each driver is taken into account to break the tie. If two alternatives are still tied with each other, the next criterion is the greater number of races in which each driver finished a race in second place and so on successively.

However, this method is not ideal as diverse problems or unsporting situations can occur as a result of the use of this system of scoring as described in [3].

As regards the constructors' championship, the first Constructors' World Championship was won by the Vanwall team in 1958. In the majority of the seasons until 1979, only the results of the best driver in the team counted towards the scoring of the championship. In the following year, there was an alteration to the rule and the points were obtained by the sum of the results of both drivers in each team, a change which has lasted until today. Only on ten occasions has the world champion constructor's team not had one of their drivers win the title of world champion. Thus, as in the case of the drivers' championship, the winning team in the constructors' championship is the team which obtains the greatest number of points, adding together the number of points obtained by their two drivers in each race of the championship. It should be noted that the sum of points can cause similar distortions to those pointed out by [3] for the drivers' championship. These distortions can be aggravated by the fact that, currently, a small team (STR-Renault) is, in practice, a branch of one of the larger teams (Red Bull Racing-Renault). In order to mitigate these distortions an adaptation of the Condorcet method for the team championship will be shown.

It should be observed that, as each team (alternative) has two drivers (sub-alternatives), this championship is characterized by the fact that each decision maker (race) can only be weakly rational.

4. Analysis of the 2013 Championship

The 2013 championship was characterised by the clear superiority achieved by the Red Bull Racing team, which finished the championship with a lead of more than 200 points from the second placed team, and for the close competition between the McLaren-Mercedes, Ferrari and Lotus-Renault teams for this second place spot.

Sebastian Vettel, the number one Red Bull Racing driver, won 13 races out of a total of 19, justifying the number of points achieved by the team in 2013. Meanwhile, the competition for second place was fiercely contested between McLaren-Mercedes, Ferrari and Lotus-Renault Renault, with a very small difference separating them, only 6 points out of 350.

The aim of this study was to obtain a ranking for the constructors' championship independent of irrelevant alternatives. As each team has two drivers in each race, it is characterised as an evaluation of alternatives with sub-alternatives and is therefore weakly rational. Thus, the analysis by the Condorcet method proposed in this article has small differences in relation to the original method.

To construct the Condorcet matrix, the alternatives are compared two by two to establish the preferences of the decision maker. In order to evaluate the preference between the teams which participate in the Formula 1 constructors' championship, it is necessary to compare the sub-alternatives of each of them, as each team participates in the championship with two drivers. Table 3 presents the comparison between the Sauber-Ferrari and Williams-Renault teams. In this table, the first column presents each of the championship races. Columns 2 to 5 present the positions of each of the drivers of these teams at the end of each of the races. For the purposes of the construction of the matrix, when comparing two teams, it was considered that in the case of any driver abandoning a race, the driver who completed most laps would have a better classification than a driver who completed fewer laps. In addition, a driver who abandoned a race would outrank another who did not classify for the starting grid, and this driver in turn was better than one who did not participate in the practice days. It is worth pointing out that that FIA (Fédération Internationale de l'Automobile) already uses the same criteria to present the results of each race. A situation which can arise with the application of the Condorcet method instead of the official method is that a driver could remain on the track driving slowly for various laps in order to improve the position of his team. If this happens it should be remembered that the race steward has the power to show the black flag (exclusion from the race) to a driver whose attitudes place him or others at risk. Columns 6 to 8 present the preference relations between the teams, where the number 1 represents the decision maker's preference, that is, that the alternative in the row is preferable to the alternative in the column.

Basically, the best placed driver in the first team is compared with the best placed driver in the second team, and the worst placed driver in the first team is compared with the worst placed driver in the second team. It is considered to be a victory for one team over another when the first is preferable over the other according to Definition 1. Similarly, the two teams are considered tied when one is indifferent to the other according to Definition 2.

Using a practical example, the race held in Malaysia can be observed in the table. Nico Hülkenberg of Sauber-Ferrari finished in eighth place, while the best Williams-Renault driver (Valtteri Bottas) finished in 11th place. Esteban Gutierrez of Sauber-Ferrari also finished ahead of Pastor Maldonado, the Williams-Renault driver. In this case, the first team is preferred in relation to the second. It is important to remember that there is no pre-determined first or second driver; this denomination is made according to the position in which the driver finishes in each race separately.

In the race held in Canada, the inverse occurred. This time the best placed driver from the Sauber-Ferrari team was Esteban Gutierrez, who finished in 20th place, while Valtteri Bottas of the Williams-Renault team finished in 14th place. Both Valtteri Bottas and Pastor Maldonado finished in positions in front of the drivers from the Sauber-Ferrari team, therefore, the Williams-Renault team is preferred.

A third situation occurred in the race in England. Nico Hülkenberg, the driver from the Sauber-Ferrari team, finished in front of the drivers from the Williams-Renault team, however, Esteban Gutierrez, also from the Sauber-Ferrari team, finished behind the drivers from the Williams-Renault team. In this case, no team is preferred, characterizing a tie between the alternatives. It is worthwhile pointing out that there will always be a tie between the alternatives when one driver from a team wins the race and the other driver does not leave the grid, in relation to any other team in which the two drivers complete at least one lap of the race: a team will be preferred in one sub-alternative but not preferred in the other.

Table 3 – Comparison between the Sauber-Ferrari and Williams-Renault teams.

Team		Sauber-Ferrari		Williams-Renault		Winning Alternative		
Driver		Nico Hülkenberg	Esteban Gutierrez	Pastor Maldonado	Valtteri Bottas	Sauber-Ferrari	Williams-Renault	Tie
Race	Australia	22	13	21	14			1

Malaysia	8	12	19	11	1		
China	10	22	14	13			1
Bahrain	12	18	11	14		1	
Spain	15	11	14	16	1		
Monaco	11	13	20	12	1		
Canada	21	20	16	14		1	
England	10	14	11	12			1
Germany	10	14	15	16	1		
Hungary	11	21	10	20		1	
Belgium	13	14	17	15	1		
Italy	5	13	14	15	1		
Singapore	9	12	11	13	1		
Korea	4	11	13	12	1		
Japan	6	7	16	17	1		
India	19	15	12	16		1	
Abu Dhabi	14	13	11	15			1
USA	6	13	17	8	1		
Brazil	8	12	16	21	1		
Total					11	4	4

After comparing the sub-alternatives in all the races, the victories of each of the alternatives and the ties are added up. The alternative which has the greater total is the preferred alternative and this result is shown by the number 1 on the row referring to the alternative in Table 4.

All of the teams are compared with each other through pairwise comparisons, for example, Ferrari is compared with the other ten teams, Mercedes is compared with Ferrari and the other nine teams which are left and so forth successively. This makes 55 comparisons, the result of the pairwise comparison of the 11 teams.

Table 4 shows the adjacency matrix obtained by the Condorcet graph for the 2013 Formula 1 Constructors' Championship. The number 1 signifies that the team indicated on the row obtained a better classification more often than the team indicated in the column. The spaces left blank are equivalent to zeros. The order in which the teams appear in the matrix is equal to their official ranking in the championship.

The final rankings of all the races were obtained from the site http://www.f1.com.

Table 4 – Adjacency Matrix of the Condorcet graph for the 2013 Constructors' Championship.

	Red Bull Racing-Renault	Mercedes	Ferrari	Lotus-Renault	McLaren-Mercedes	Force India-Mercedes	Sauber-Ferrari	STR-Ferrari	Williams-Renault	Marussia-Cosworth	Caterham-Renault
Red Bull Racing-Renault		1	1	1	1	1	1	1	1	1	1

Mercedes			1	1	1	1	1	1	1	1	1
Ferrari				1	1	1	1	1	1	1	1
Lotus-Renault					1	1	1	1	1	1	1
McLaren-Mercedes						1	1	1	1	1	1
Force India-Mercedes							1	1	1	1	1
Sauber-Ferrari								1	1	1	1
STR-Ferrari									1	1	1
Williams-Renault										1	1
Marussia-Cosworth											
Caterham-Renault										1	

In order to extract a ranking from the matrix one begins by performing a descending distillation (DIAS ET AL., 1996). In order to do this, one observes if there is any team which outranks all the others, in other words, if there is any row on which the only zero is on the principal diagonal. This team is removed and the procedure is repeated.

Using the example of the first distillation to be performed, it is observed that the Red Bull Racing-Renault team only has a zero in the column which corresponds to itself, therefore the rows and columns which relate to it are removed from the matrix, leaving Table 5 as shown below.

Table 5 – Second Adjacency Matrix from the Condorcet graph for the 2013 Constructors' Championship.

	Mercedes	Ferrari	Lotus-Renault	McLaren-Mercedes	Force India-Mercedes	Sauber-Ferrari	STR-Ferrari	Williams-Renault	Marussia-Cosworth	Caterham-Renault
Mercedes		1	1	1	1	1	1	1	1	1
Ferrari			1	1	1	1	1	1	1	1
Lotus-Renault				1	1	1	1	1	1	1
McLaren-Mercedes					1	1	1	1	1	1
Force India-Mercedes						1	1	1	1	1
Sauber-Ferrari							1	1	1	1
STR-Ferrari								1	1	1
Williams-Renault									1	1
Marussia-Cosworth										
Caterham-Renault									1	

The same procedure will then be carried out, which will successively exclude Mercedes, Ferrari, Lotus-Renault, McLaren-Mercedes, Force India Mercedes, Sauber Ferrari, STR- Ferrari (Toro Rosso), Williams-Renault, Caterham-Renault and Marussia-Cosworth.

With this distillation, it is possible to obtain a new ranking of all the teams. It should be noted that there was only one alteration in relation to the official classification, that concerning the Caterham-Renault and Marussia-Cosworth teams. All the other teams maintained the same ranking as in the official ranking.

If it is not possible to make a decreasing ranking of all the teams as, from a certain point, there are no dominant teams, then the inverse procedure should be carried out: an ascending distillation. In this type of distillation, it is possible to identify the dominated teams.

The ranking obtained by the Condorcet method compared with the official ranking is shown in Table 6.

Table 6 – Final results for 2013 (official and Condorcet).

Team	Condorcet Ranking	Official Ranking	Variation
Red Bull Racing-Renault	1	1	0
Mercedes	2	2	0
Ferrari	3	3	0
Lotus-Renault	4	4	0
McLaren-Mercedes	5	5	0
Force India-Mercedes	6	6	0
Sauber-Ferrari	7	7	0
STR-Ferrari	8	8	0
Williams-Renault	9	9	0
Caterham-Renault	10	11	+1
Marussia-Cosworth	11	10	-1

5. Conclusions

During this article it has been shown that, due to the formal analogy between the Formula 1 World Championship and a multi-decision maker selection process, there is no regulation which can be considered fair.

However, the regulation in force until 2002 intensified the disadvantages of the Borda method on which it was based. The Condorcet method permits one to get around the distortions of the variant of the Borda method used, but does not always supply a complete ranking, owing to the existence of cycles of intransitivity. In addition to this, it is an extremely technical method to be understood by the general public.

The comparison of the official result and that obtained by the Condorcet method shows very similar results, with variations only between the Caterham-Renault and Marussia-Cosworth teams.

Using the results of the 2013 championship, it was possible to rank all the teams through the Condorcet method, as there was no cycle of intransitivity among the alternatives. When these intransitive cycles arise, the solution that it supplies is less sensitive to the irrelevant alternatives than the Borda method and an alternative is to use the Copeland method [5]. This method consists of counting how many times each alternative was preferable to the others. The alternatives are then ranked by the result of this sum. The Copeland method combines the advantage of supplying a complete ranking with the fact that it gives the same result as the Condorcet method, when this does not present an intransitivity cycle. When these cycles exist, the Copeland method permits the ranking to be made and maintains the position of the alternatives (teams) which do not belong to any intransitivity cycle.

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